STARFLIGHT MECHANICS NOTES

by

The Starship Design Group

R.G. Finke, Editor

April 2001

PREFACE

This document is intended to gather in one place the answers to the questions about starflight mechanics and propulsion technologies raised by Group members in the first three years of the Starship Design project. (The emphasis is not so much on providing answers as it is on showing how to find answers.)

The intent of the documentation here has been to use only selections from notes as originally posted and to reproduce them as closely as possible, except where they were incorrect or incomplete, or where they were inconsistent with other postings. (A few useful facts have been added here and there to make the write-up more comprehensive.) It was of primary interest to capture and preserve the feeling of discovery that accompanied the interactions of the members.

Creator of the Lunar Institute of Technology / Starship Design Project (and host/ coordinator for the first year of correspondence among the members):

David Levine

Starship-Design-Group contributors to these Notes (identified by first names in the text):

Reinald (Rex) Finke (unattributed postings are by the editor)
Kevin Houston
Zenon Kulpa (Poland)
Brian Mansur
Brandon Neill
Lee Parker
Kelly Starks
Timothy van der Linden (The Netherlands)
Steve VanDevender
Ken Wharton

(The editor was a member of the Group from March 1996 to July 1997 and accessed copies of the Newsletter from its inception in August 1994 through its suspension in October 1995. He did not access members' informal correspondence between October 1995 and March 1996, or after June 1997.)

Charter for LIT Starship Design Project: (David Levine)

==========

The members of LIT are currently involved in a design project. This design project will ultimately result in the basic design of a starship and a fairly detailed plan for the mission. This will not result in the construction of a real starship and most likely never will. Most current starship designs (i.e., Orion, Daedalus, etc.) are considered good exercises and practice, but it is generally accepted that our first real starship will be a different design from any currently considered. However, it will be based upon the lessons learned in these early designs. At best, we may be able to hope for our starship to enter the "literature" alongside some of the other famous designs. Certainly ours will be the first to be designed by so large a group spread out over so large an area.

Because there are so many different people involved and because everyone has very different ideas about why we should be going to the stars, it has become necessary to give a framework for what we are doing. Not everyone may agree that this framework is the best. But for the sake of our thought experiment, I ask you to accept it for now. You are certainly free to pursue your own designs, and LIT will support you in your independent research with the efforts of our student body. Your work will also be gratefully given a place on the LIT World Wide Web pages. However, in order to avoid the usual internet chaos, please stick to this Charter when submitting items for the Newsletter. In the near future, I would like there to be a FAQ for the design project as well, so that we don't repeat the same questions time and time again. If anyone would like to help with this project, please speak up.

Here is the story: As the years progress, there will be more and more studies of nearby star systems. Most current indications seem to say that planets, on the whole, are fairly common. Most newborn stars in the Trapezium, for instance, seem to be surrounded by accretion disks. There have been many indirect observations of planets (i.e., inferences made from disturbances in the motion of nearby stars). It is likely that as our technology improves, we will eventually obtain direct evidence of planets surrounding nearby stars. Orbiting telescopes and occultation disks could be one way to obtain direct images.

Our assumption is that such an observation has been made: Surrounding many nearby star systems, large Jovians and small terrestrial planets are detected. However, spectral analysis gives no sign of interesting molecules (i.e., oxygen, water vapor, etc). However, around τ Ceti, which is 11.9 light-years distant, six planets are detected. τ Ceti itself has a mass of 0.82 times that of the Sun, 0.44 times its luminosity, and 1.67 times its radius. It is a G8 star (the Sun is G2). Out of the six detected planets, five of them are Jovians and one is terrestrial. There may be additional planets we have been unable to detect. The five Jovians are:

II: 1.16-1.18 AU, approx. 30 Earth masses, orbital period of 1.4 years
III: 1.99-2.01 AU, approx. 80 Earth masses, orbital period of 3.2 years
IV: 2.79-2.81 AU, approx. 1.30-1.34 Jupiter masses, period 5.3 years
V: 5.60 AU, approx. 0.92-0.96 Jupiter masses, orbital period 14.6 years
VI: 11.10 AU, approx. 1.05-1.09 Jupiter masses, orbital period 41.0 years

The one terrestrial planet, as well as can be determined, is:

I: 0.71-0.73 AU, approx. 0.20-0.22 Earth masses, period 8 months

Spectral analysis strongly indicates the presence of oxygen, nitrogen, carbon dioxide and water vapor. However, we cannot give relative amounts. These elements are, however, in large enough amounts to be detected. τ Ceti's "lifezone" extends from about 0.62 to 0.78 AU, so "I" looks pretty good. There have been no radio signals received from this or any other star.

The public is clamoring for a manned mission, and significant funding is available for this now. It is assumed they are not interested in waiting for results from a probe first. The reason we need to assume this is that we are trying to design this starship back here in the 1990s (remember, LIT takes place in the "future"... more on this later), and we don't have any information about our target (or any target), so we've got to come up with some kind of conditions (no matter how unlikely) that allow us to design a manned mission. A manned mission is inherently more interesting (and, of course, challenging), even if an unmanned mission is more likely and a better choice. Since we're not really building a starship, we should go all the way with our scenario and have some fun while we're at it.

So, our job is to design a manned expedition to this star system. We are fairly sure we'll find something interesting there, but not 100% sure. Are we exploring or colonizing? That depends on the energy available. If we can't get our astronauts back, we need to look seriously at outfitting them for colonization in a number of possible environments. Assume by this time we have some experience with temporary colonies (<50 people) on Mars, and (of course) slightly larger colonies on the Moon. (Speaking of which, the Moon's population in the LIT world is near 3,000, but quite spread out among a variety of government installations and private enterprises.) Also, because this public funding is coming from the public's keen interest, it is unlikely they will fund a slowship. We should try to focus on fast vessels, but if this turns out to be impossible, then we should go ahead and design a slowship.

What kind of technology are we looking at? No black holes or warp drives, please. I won't say they're impossible, but I would like to stick to the fictional background of LIT, which says that the year is 2055. In other words, what can we do, technologically, with the next 60 years? Who knows, the science of warp drives could even be developed by then, but it's unlikely in the extreme that we could engineer a starship with such technology. Remember, engineering follows science, and even if we have the science, if

we don't have the engineering, we can't do it. Fusion today is a good example of this. We pretty much know the science, but we don't know all of the engineering. For many, many years we've been told fusion is "just around the corner." Well, finally, we'll definitely have fusion in the 2050s. That's one possible power source. So is fission. So are solar sails and laser sails. As for antimatter... well, you'll have to convince everyone else here if you want to go that route. Assuming someone came up with a way to generate copious quantities of antimatter in the next few years and a rigorous power plant program was followed... it might *just* be possible. But I'II leave that for everyone to decide. What about electromagnetic launching? Or RAIRs? Or even ramjets for that matter. Do you think we could construct the huge electromagnetic fields required to form the scoop?

Let's see... what else? We probably will have some kind of permanent orbital presence, and a permanent presence on the Moon (though likely small). However, we may not have "orbital shipyards." Therefore, if we're building the spaceship in orbit, we have to build the facilities, too. However, by this time this probably won't be that bad, because we'll have some kind of construction experience in orbit from space stations, etc. Besides, building in orbit or on the Moon is cheaper than on the Earth just due to the lifting costs.

Anyway, this should give us a general framework. Let's try to keep the discussions on-topic. If you've got anything you feel should be added to this, please say so, and we'll all consider it.

==========

CONTENTS

Preface.. 2

Charter for LIT Starship Design Project 3

I. INTRODUCTION 7

II. OPTIMUM ANTIMATTER-POWERED ROCKETS 17

A. Conversion Efficiency 100% 17
B. Arbitrary Conversion Efficiency 21

III. PROTONS vs. ELECTRONS for Relativistic Electric Thrusters 25

A. Questions Regarding Relativistic Electric Thrusters 25
B. Accelerated Protons 27
C. Accelerated Electrons 28
D. Comparison 31

IV. THE SIZE OF THE PROBLEM 33

V. MOTION OF BEAM-DRIVEN SAIL 41

A. Power Requirements for Constant Acceleration of Sail 41
B. Acceleration of Sail Driven by Constant-Power Beam 43
C. Deceleration of Sail Pushed by Constant-Power Beam 48
D. Current for Electric Retro-Thruster for Sail 53
E. Required Microwave-Antenna Size 54
F. Beam Properties, Control, etc 57

ADDENDUM: Useful Constants 62

ADDENDUM: Stars Within 16 Light-Years of the Sun 63

ADDENDUM: Fortran Programs Used to Perform the Calculations 65

APPENDIX: An Engineering Review of Relativity for Interstellar Flight A-1

I. INTRODUCTION (Bits and Pieces: Prerequisite ideas and ancillary issues)

On 2/6/95 Kelly wrote–
"If we could get a ship to relativistic speeds, most all of our debates (ship design, artificial-g systems, food, etc..) would be turned upside down."

On 3/27/95 Kelly wrote–
"Everything else depends on the drive systems. Until we know how fast, how big, how far..., a lot of other threads tend to get overwhelmed."

(posted 3/18/96)
On 3/15/96 Brian wrote (to Kelly) that it is "so frustrating being stuck in the 1990s while trying to design a ship whose technology is really at least 100 years beyond our reach."

In an IDA study of future space launch vehicles conducted some 30 years ago, the purpose was (paraphrased)–
(1) To make an internally consistent comparison of the different options, with conceptual design only in enough detail to allow rating them regarding feasibility and cost, and
(2) To determine the advancements in technology in relation to current levels which are required to make them achievable.
While that study mainly considered options consistent with a decision to develop the Space Shuttle only seven years hence (which we didn't know at that time), it spent a lot of time examining the competitive standing and technological requirements for scramjets (aka "the Aerospaceplane"), which even today may still be decades in the future (if ever).

Such a comparative study of starship propulsion systems could be a guide to steer future efforts away from the losers, but mainly would provide the "mission push" to support advancement in the key technologies.

(posted 4/25/96 to Brandon)
Regarding "apparent" and "proper" velocities, let me risk being too long-winded in order to try to be precise–

Any property (e.g., distance or time) measured in the frame of reference at rest with respect to the property is termed a "proper" value. Any property measured from a frame of reference that has a relative velocity with respect to the property being measured is termed an "apparent" value. The starmap distance is a "proper" distance. (The contracted distance between stars measured by a moving starship observer is an "apparent" distance.) The trip time measured by an ordinary clock on the control panel of the starship is a "proper" time. (The trip time measured by two clocks, one at the start and one at the finish, at rest with respect to the Earth but "moving backwards" with respect to the starship, is an "apparent" time.) "Apparent" velocity, which I call V, is either an apparent

distance divided by a proper time or a proper distance divided by an apparent time. "Proper" velocity, which I call U, is a proper distance divided by a proper time. While V (in lt-yr/yr) saturates at the value 1.0, U can take on any value from zero to infinity. (This terminology can be found in a number of relativity books. However, calculations are more often made of the apparent velocity, so the proper velocity is not as familiar.)

(posted 4/3/96 to Timothy)
Why do I prefer proper velocity to apparent velocity? If it isn't enough that the proper velocity is the velocity the crew cares about (it's a direct determinant of how long the trip will take them), the proper velocity is more nearly linear with effort to achieve it and it is more nearly linear with time to reach it, than is apparent velocity. It gives results more nearly equivalent to Newtonian mechanics, validating intuition. In the following table I give the mass ratio to achieve various velocities (for the exhaust velocity 0.8516, which is the optimum for a final proper velocity of 5; see Section II-A-3), the proper time to reach the velocity at an acceleration of 1 g (1.0324 lt-yr/yr^2) and the apparent time (in yr) at 1 g.

proper vel (lt-yr/yr)	apparent vel (lt-yr/yr)	mass ratio	proper time (yr)	apparent time (yr)
1	0.707	2.81	0.854	0.969
2	0.894	5.45	1.398	1.937
3	0.949	8.46	1.761	2.906
4	0.970	11.70	2.029	3.874
5	0.981	15.11	2.240	4.843

(posted 5/16/96)
Beware of using chemical-rocket parameters for fusion or antimatter rockets. The parameter to be most wary of is the specific impulse, "I_{sp}." I_{sp} is the rocket-engine thrust per unit mass flow rate (out of the nozzle).

We are interested in an equation that gives the velocity increment, "ΔV," of the rocket stage in terms of the ratio of the initial mass of the stage to the final mass, the "mass ratio". The equation is called the "rocket equation" in the West and the "Tsiolkovsky equation" in Russia (after the first person to derive it). The depletion in mass (initial mass minus final mass) for a chemical rocket stage is solely propellant. For a fusion/antimatter rocket, however, the mass depletion is the sum of the propellant mass (out of the "nozzle," or accelerator) and the mass converted to energy in the fusion reaction or the matter/antimatter-annihilation reaction.

For a chemical rocket stage, the parameter that links the ΔV with the function of the mass ratio (the natural log) is simply the exhaust velocity, V_{exh}, i.e.,

$$\Delta V = V_{exh} \ln(\text{mass ratio}) \quad .$$

I_{sp} was invented, I believe, to allow English-system (foot-pound-second) engineers to talk about rocket performance with metric-system (meter-kilogram-second) engineers by "non-

dimensionalizing" the exhaust velocity. This was done by dividing it by the constant g_c, i.e.,

$$V_{exh}/g_c = I_{sp} \quad .$$

This reduces the units to "seconds", which are common to both systems. The constant g_c is only incidentally equal in value to the standard acceleration of gravity; it is properly referred to as the conversion factor from mass to force units, either 32.17405 lbmass-ft/(sec^2-lbforce) or 9.80665 kgmass-m/(sec^2-kgforce). (To be totally correct, the units of I_{sp} should be lbforce-sec/lbmass or kgforce-sec/kgmass, but the ratios lbforce/lbmass and kgforce/kgmass are usually just left out of both I_{sp} and g_c.)

For a fusion or antimatter rocket, additional parameters that must be included in the rocket equation are the ratio of the fusion/antimatter mass to the mass converted to reaction energy (the mass/energy ratio "f") and the efficiency of conversion of reaction energy to exhaust kinetic energy (η). Also, the velocity increment must be put into relativistic terms. The relativistic fusion/antimatter rocket equation becomes much more complicated than the chemical rocket equation.

(posted 3/23/96 to Timothy)
My derivation of the relativistic rocket equation for fusion or antimatter rockets with arbitrary efficiency of conversion of reaction energy to exhaust energy is as follows:
Define:
V_{exh} = exhaust velocity;
V_{end} = velocity at end of burn = ΔV (for zero initial velocity);
γ_{exh} = relativistic energy factor of exhaust = $1\Big/\sqrt{1-(V_{exh}/c)^2}$;
f = mass/energy ratio = ratio of sum of reaction-particle mass/energies to resulting energy;
η = conversion efficiency from reaction ("resulting") energy to exhaust kinetic energy.
Let us call rate of change of mass, dM/dt' = $\dot{M}$ (a common notation); where
M = mass of ship at any time, and t' = time read on ship's clock;
M_i = initial mass;
M_{bo} = burnout mass;
M_e = mass of exhaust (propellant); and
M_n = mass of nuclear (fusion or annihilation) fuel. (Note: all masses are rest mass.)

$$\text{Exhaust power} = (\gamma_{exh} - 1)\ \dot{M}_e c^2 = \eta\ (\dot{M}_n/f)\ c^2 \quad ,$$

so,

$$\dot{M}_n = (\gamma_{exh} - 1)\ (f/\eta)\ \dot{M}_e \quad .$$

For matter/antimatter annihilation (f = 1), the exhaust mass comes from a reservoir additional to the matter/antimatter reservoir, so–

$$\dot{M} = \dot{M}_e + \dot{M}_n \quad .$$

For fusion, the exhaust mass is composed of part of the fusion reaction products, with the rest dumped at "zero" exhaust velocity, so–

$\dot{M} = \dot{M}_n$.

Combining the two $\dot{M}$ relations above gives–

$\dot{M} = [\{1\} + (\gamma_{exh} - 1)\ (f/\eta)]\ \dot{M}_e$,

where the additive {1} is left out if f >>1 (fusion).

The "velocity parameter equation of motion" (θ is the "velocity parameter," defined in an equation later, and F is the thrust, measured in the frame of reference of the ship) is:

$F = M\ c\ (d\theta/dt')$,

but

$F = \dot{M}_e\ \gamma_{exh}\ V_{exh}$.

Setting the two expressions on the right equal and solving for $\dot{M}_e$ gives

$\dot{M}_e = M\ c\ (d\theta/dt')/(\gamma_{exh}\ V_{exh})$ (ignoring minus sign).

So, from the equation for $\dot{M}$ above, we get

$\dot{M} = dM/dt' = [\{1\} + (\gamma_{exh} - 1)\ (f/\eta)]\ M\ c\ (d\theta/dt')/(\gamma_{exh}\ V_{exh})$.

Canceling the dt' and dividing both sides by M gives

$dM/M = [\{1\} + (\gamma_{exh} - 1)\ (f/\eta)][c/(\gamma_{exh}\ V_{exh})]\ d\theta$.

Integrating M from M_i (= M[0]) to M_{bo}, while θ increases from 0 to Δθ, gives

$\ln(M_i/M_{bo}) = [\{1\} + (\gamma_{exh} - 1)\ (f/\eta)][c/(\gamma_{exh}\ V_{exh})]\ \Delta\theta$,

or

$\Delta\theta = (\gamma_{exh}\ V_{exh}/c)/[\{1\} + (\gamma_{exh} - 1)(f/\eta)]\ \ln(M_i/M_{bo})$.

The definition of the velocity parameter Δθ is

$\Delta V = c \tanh \Delta\theta$, [and $\Delta U = c \sinh \Delta\theta$],

so the relativistic rocket equation (giving the increment in apparent velocity) becomes

$\Delta V = c \tanh[(\gamma_{exh}\ /[\{1\} + (\gamma_{exh} - 1)\ (f/\eta])\ (V_{exh}/c)\ \ln(M_i/M_{bo})]$,

(return to 5/16/96 post)

or the increment in proper velocity is given by

$\Delta U = c \sinh[(\gamma_{exh}\ /[\{1\} + (\gamma_{exh} - 1)\ (f/\eta)])\ (V_{exh}/c)\ \ln(M_i/M_{bo})]$

(Note: for fusion, i.e., the mass/energy ratio f greater than 1, the {1} term does not appear because some of the fusion reaction products can be used as propellant.)

In the case where f = 1 (matter/antimatter annihilation), the relativistic rocket equation becomes

$\Delta U/c = \sinh[(\gamma_{exh}\ /[1 + (\gamma_{exh} - 1)/\eta]\ (V_{exh}/c)\ \ln(M_i/M_{bo})]$.

In the simplistic antimatter-rocket case where η = 1 (100 percent conversion of reaction energy to exhaust kinetic energy), the relativistic rocket equation reduces to

$\Delta U/c = \sinh[(V_{exh}/c) \ln(M_i/M_{bo})]$

in a form somewhat similar to the chemical rocket equation. (Note that all velocities are non-dimensionalized now by dividing them by c.)
I hope this properly expresses the shortcomings of using chemical-rocket performance relations for fusion/antimatter rockets.

On 10/6/96 Timothy wrote–
"...how [could you] prove that definition of $\theta = a\,t'$?"

(posted 10/8/96 to Timothy and Steve)
The basis for the equation $\theta = a\,t'$ can be determined if one turns to my derivation of the "velocity-parameter equation of motion" ("VPEM") on p. 14 of the Appendix.

One could find a derivation of the VPEM in the book "Spacetime Physics" by Taylor and Wheeler [1966], if it were still available (it has been superseded by a second edition, where this derivation no longer appears).

(posted 10/8/96 to Steve and Timothy)
To quote selectively page 97 of Taylor and Wheeler:
"[Exercise] *51. Clock paradox III*
...
"(b) How much velocity does the spaceship have after a given time? *This is the moment to object to the question and to rephrase it.* Velocity β *is not the simple quantity to analyze. The simple quantity is the* velocity parameter θ. *It is simple because it is* additive *in this sense: Let the velocity parameter of the spaceship in Figure 76 with respect to the imaginary instantaneously comoving inertial frame change from 0 to* $d\theta$ *in an astronaut time* $d\tau$ [= "proper time" dt′, in my notation]. *Then the velocity parameter of the spaceship with respect to the* laboratory *frame changes in the same astronaut time from the initial value* θ *to the subsequent value* $\theta + d\theta$. *Now relate* $d\theta$ *to the acceleration* [g/c] *in the instantaneously comoving inertial frame. In this frame*

$[g/c]\, d[t'] = d\beta = \tanh d\theta \sim d\theta$ [Note the approximation (~) invoked here.]

so that

$d\theta = [g/c]\, d[t']$. *(64)"*

---

g in lt-yr/yr^2 and c in lt-yr/yr

A variant of this last equation with g replaced by F/m–

$F = m\ c\ d\theta/dt'$

is what I call the "velocity-parameter equation of motion ('VPEM')." The authors do not use that name. I believe that my derivation of the VPEM in the Appendix is more straightforward because it doesn't need the approximation(s) in the sequence above.

On 5/3/96 Kelly wrote–
"I have a table where I list various fusion fuel cycles. It lists the resulting energy in MeV. For those of us who aren't familiar with translating MeV into anything, could someone tell me what the speed of the resulting particles is?"

(significantly revised posting of 5/16/96)
The resulting energy is shared between two (or more) resulting particles, apportioned according to conservation of momentum in the frame of reference of the initiating particles.

The relation that gives the velocity as a function of energy of each particle is derived from–

kinetic energy = MeV = $(\gamma - 1)\, m\, c^2$

so $\gamma = 1 + \text{MeV}/mc^2$

but $\gamma = 1\Big/\sqrt{1-\beta^2}$ (where $\beta = v/c$)

so $\beta = \sqrt{1 - 1\Big/\left(1 + \text{MeV}\Big/mc^2\right)^2}$.

For protons, mc^2 = 938.3 MeV; for deuterons, essentially twice that; for tritons, three times; and for alphas, four times. For electrons, mc^2 = 0.511 MeV.
A table of values of β as a function of energy for the five particles is given below–

Energy	β				
(MeV)	protons	deuterons	tritons	alphas	electrons
10	.1448	.1028	.0841	.0729	.9988
20	.2032	.1448	.1186	.1028	.9997
50	.3140	.2263	.1860	.1616	.9999
100	.4282	.3140	.2597	.2263	1.0000
200	.5662	.4282	.3582	.3140	1.0000
500	.7579	.6136	.5281	.4704	1.0000
1,000	.8750	.7579	.6749	.6136	1.0000
2,000	.9476	.8750	.8113	.7579	1.0000

In the calculation of the apportionment of energy in a collision between two initiating particles, one must normally consider two different coordinate systems: the laboratory system, in which the bombarding (usually the lighter) particle is moving at high speed and the target (usually the heavier) nucleus is at rest, and the center-of-mass system, in which the reaction energy is apportioned between the reaction products so as to give them equal and opposite momentum after the reaction. In a thermonuclear (fusion) reaction, however, the initial momentums of the reacting particles are negligibly small in comparison with the momentums after the reaction, so the center of mass is essentially at rest and analysis in the center-of-mass system is adequate, as follows (non-relativistically)–

Let's define the following quantities–

m = mass of lighter reaction product, in atomic mass units ("amu")
(e.g., mass of proton = 1.007 276 470 amu [1 amu = 931.5 MeV])
M = mass of heavier reaction product, in amu
E = "resulting energy" in MeV of reaction products
(this is with respect to their center of mass)
$= [(m_1 + M_1) - (m_2 + M_2)]\ 931.5$ MeV
(where sub-1 designates particles before the reaction and sub-2 designates particles after the reaction)
v = velocity of the lighter reaction product with respect to the center of mass
V = velocity of the heavier reaction product with respect to the center of mass.

After the reaction, the momentums of the two particles are equal and opposite (*and in a random direction*), and the sum of the kinetic energies of the particles is equal to the "resulting energy," i.e.,

$$m\,v = M\,V$$
$$\tfrac{1}{2}\,m\,v^2 + \tfrac{1}{2}\,M\,V^2 = E \quad .$$

Thence–

$$\tfrac{1}{2}\,m\,v^2 + \tfrac{1}{2}\,M\,(m\,v/M)^2 = E$$
$$\tfrac{1}{2}\,m\,v^2\,[1 + (m/M)] = E$$
$$\tfrac{1}{2}\,m\,v^2 = E_1 = E\,M/(M + m) \quad ,$$

and similarly–

$$\tfrac{1}{2}\,M\,V^2 = E_2 = E\,m/(M + m) \quad ,$$

for which the corresponding velocities can be found from the β relation above.

(The calculations [except for E] can be simplified without appreciable loss of accuracy by considering only integer numbers of amu, e.g., 1 instead of 1.007276 for a proton.)

My copy of the U.S. Naval Research Laboratory "Plasma Formulary" (1987 Revision) gives the energy apportionment for the following reactions, and gives the reaction rates in cm^3/sec (the reaction cross-section times the plasma particle velocity; a relative measure of the ease of producing the reaction) for only the first five reactions at a plasma temperature of 1 keV–

				Reaction rate
(1a)	$D + D$	——> T (1.01 MeV) + p (3.02 MeV)		
	50%		(1a + 1b)	1.5×10^{-22}
(1b)		——> He^3 (0.82 MeV) + n (2.45 MeV)		
	50%			
(2)	$D + T$	——> He^4 (3.5 MeV) + n (14.1 MeV)		5.5×10^{-21}
(3)	$D + He^3$	——> He^4 (3.6 MeV) + p (14.7 MeV)		10^{-26}
(4)	$T + T$	——> He^4 + 2 n + 11.3 MeV		3.3×10^{-22}
(5a)	$T + He^3$	——> He^4 + p + n + 12.1 MeV		
	51%			
(5b)		——> He^4 (4.8 MeV) + D (9.5 MeV)		
	43%		(5a + 5b + 5c)	10^{-28}
(5c)		——> He^5 (2.4 MeV) + p (11.9 MeV)		
	6%			
(6)	$p + Li^6$	——> He^4 (1.7 MeV) + He^3 (2.3 MeV)		
(7a)	$p + Li^7$	——> 2 He^4 + 17.3 MeV		
	20%			
(7b)		——> Be^7 + n – 1.6 MeV		
	80%			
(8)	$D + Li^6$	——> 2 He^4 + 22.4 MeV		
(9)	$p + B^{11}$	——> 3 He^4 + 8.7 MeV		
(10)	$n + Li^6$	——> He^4 (2.1 MeV) + T (2.7 MeV)		

Note that the reactions involving He^{3*} are at least four orders of magnitude more difficult to produce than the the reactions involving D and T at a plasma temperature of 1 keV, the lowest temperature given in the table of reaction rates. (The $D + He^3$ reaction rate at 5 keV given in the PF is about the same as the D + T reaction rate at 1 keV shown here.)

(posted 6/16/96 to Timothy)
The Coulomb barrier is twice as high for the $D + He^3$ reaction as it is for the D + T reaction (which is about 660 keV by my calculation), so you'd expect the tunneling at 1 keV indeed to be much less for $D + He^3$.

---

* He^3 is 1.37×10^{-4} percent of helium; i.e., there is about 10^{-6} kg He^3 per kg helium. In 1984, world reserves of helium were estimated to be about 300 billion cu. ft. At 5.05 gm/ft^3 helium, 300 billion cu. ft. is 1.515×10^6 metric tons of helium, which gives about 1,500 kg of He^3 as the world supply. This amount of He^3, coupled with the appropriate amount of D_2 in the thermonuclear reaction above, would yield about 28 thousand Mw-yr of energy, or about one tenth of the electrical energy produced by the US in the year 1987 (see Section IV).

.
(posted 6/22/96 to Timothy)
The reaction rate is a measure of the probability that the reaction will take place for a given plasma temperature and pressure. It's also a measure of the power out. If it can also be considered as a measure of the size of the fusing chamber, then the fusing chamber for a D + He^3 reaction must be 10^5 times as large as one for a D + T reaction (at the same pressure, for a plasma temperature of 1 keV).

(posted 5/7/96 to Timothy)
On 5/6/96 Timothy wrote–
>Rex has given a 2-particle calculation but I wonder if that makes much sense, since all >energy/momentum is finally directed in one direction namely backwards..."

When the reaction products emerge from the compound nucleus (the clump of nucleons made up of the initial particles), they do not know which direction is "backwards." What forces act on the reaction products to make them move in the preferred direction? The momentum of their center of mass must be conserved unless external forces act.

(posted 5/12/96 to Timothy)
The thermonuclear plasma is composed not only of reaction products, but also of fuel ions (and electrons). The MeV-level initial energy of the reaction products should not be given as exhaust velocity to exhaust particles. It is needed to maintain, through moderating (energy-transfer) collisions, the fuel-ion/electron plasma at the required million-degree temperature (approximately 100 eV, according to Boltzmann's constant, which is 11,605 degrees per eV), against the continual drain of energy through thermal radiation. The plasma couldn't stand the additional losses it would incur if the plasma touched the wall. These are not molecules that bounce off the walls; they are nuclear particles that are scattered predominantly forward (with respect to their velocity before contact with the wall) into the wall and are absorbed. The plasma is kept isolated from the wall in some sort of electromagnetic "bottle."

(posted 5/25/96 to Timothy, quoting his 5/23 letter to me)
Do particles exit a Tokamak? There are probably parasitic mechanisms that bleed off pressure. Maintaining pressure in a thermonucler plasma is not as easy as maintaining pressure in a chemical rocket engine. Consider the relative magnitudes of pressure and the relative penetrability of the "walls."

>But once that temperature has been built up, the amount of particles that go in should >also come out.

They may come out in an uncontrollable way, e.g., as leakage in unproductive directions.

>The total (kinetic) energy of the particles that come out should equal the energy >produced (minus the energy losses) while they reacted.

Ah, there you have the key: "minus the energy losses." Have you considered the T^4 losses from a plasma where the T is a million degrees? The T^4 losses don't leave much kinetic energy for exhaust particles; most of the original kinetic energy of the reaction products is used up in collisions maintaining the temperature of the plasma. The central problem of thermonuclear power is to get the reaction's energy-production rate going as fast as the radiation losses. The exhaust particles (if any) should almost certainly not have much more energy than the average plasma ion (i.e., of the order of 100 eV).

II. OPTIMUM ANTIMATTER-POWERED ROCKETS (Minimum Antimatter Fuel)

On 3/12/96 Kelly copied from his web site to me–
"... antimatter can be destroyed to create tremendous amounts of energy. Pound for pound an antimatter/matter reaction releases over a hundred times as much energy as a fusion reaction. Unfortunately, though it releases more energy, this energy is harder to use directly to power the ship, and it is far more dangerous to handle. If we could synthesize the tens of thousands of tons of antimatter this would take, it would have the potential of exploding with a force of hundreds of millions of H-bombs. We do not have the technology needed to synthesize, store or ship antimatter on this scale, and are not likely to get it by 2050."

(posted 4/4/96)
A. CONVERSION EFFICIENCY 100%

1. INTRODUCTION

It was first pointed out in this forum by Timothy in his calc.txt of 2/96 that there is an optimum ratio of exhaust velocity to final rocket velocity relativistically. The existence of this optimum indicates that there is a minimum in the amount of antimatter fuel required to accelerate a starship to any given final mission velocity.

This Section provides the numbers that show how the ratio of the minimum antimatter mass to initial starship mass varies with the desired mission velocity at the end of the first, i.e., acceleration, burn for the simple case in which the efficiency of conversion of matter/antimatter annihilation energy to exhaust kinetic energy is 100%. Consideration of antimatter conversion efficiencies less than 100% is treated in Section II-B. Consideration of provision for dumping particles of the opposite charge to the exhaust particles to maintain charge neutralization is treated in Section III.

2. ANALYSIS (adapted and expanded from Timothy's calc.txt)

We define the following operative quantities:

V = "apparent" velocity = starmap distance/Earth time, in lt-yr/yr (note: c = 1 lt-yr/yr),
U = "proper" velocity = starmap distance/starship time, in lt-yr/yr,
V_{end} and U_{end} are the velocities at the end of the acceleration burn (at "burnout"),
V_{exh} and U_{exh} are the exhaust velocities,
γ = relativistic energy factor = $1 \Big/ \sqrt{\left(1 - V^2\right)}$,
$U = \gamma V$,
$V = U \Big/ \sqrt{\left(1 + U^2\right)}$,
γ_{end} = energy factor for V_{end}
γ_{exh} = energy factor for V_{exh}

M = starship mass (= M_i initially; = M_{bo} at burnout),
r = starship mass ratio = M_i/M_{bo}
M_a = annihilation mass used during acceleration burn for relativistic rocket
= twice the mass of antimatter = 2 M_{am} and
M_p = mass of propellant used during acceleration burn for nonrelativistic and relativistic rockets.
The propulsive energy efficiency (let's call it eff) is the ratio of the final vehicle kinetic energy to the total exhaust kinetic energy.

Non-relativistically–

final vehicle energy = $\frac{1}{2} M_{bo} V_{end}^2$
total exhaust energy = $\frac{1}{2} M_p V_{exh}^2$
eff = $(M_{bo}/M_p)(V_{end}/V_{exh})^2$

Now from the rocket equation
M_i/M_{bo} [= $(M_{bo} + M_p)/M_{bo}$] = $\exp(V_{end}/V_{exh})$
we get
$M_p/M_{bo} = \exp(V_{end}/V_{exh}) - 1$.

If we set $x = V_{end}/V_{exh}$ to simplify, we get for the energy efficiency the expression
eff = $x^2/[e^x - 1]$.

This has a maximum value 0.6476 for x = 1.5936.

So, if the burnout velocity of a non-relativistic rocket is 1.59 times its exhaust velocity, the energy efficiency is a maximum of 64.8 percent. That is, the final vehicle energy can be no greater than 64.8 percent of the exhaust energy. This limitation is not an important consideration for a non-relativistic chemical rocket because in that case energy is a negligible component of mass depletion.

Relativistically, for 100% conversion of annihilation energy to exhaust kinetic energy–

final vehicle kinetic energy = $M_{bo} (\gamma_{end} - 1) c^2$
total exhaust kinetic energy = $M_p (\gamma_{exh} - 1) c^2 = M_{am} c^2$
which gives $M_p = M_{am}/(\gamma_{exh} - 1)$ ...(no energy losses)
but relativistically $M_p = M_i - M_{bo} - M_{am}$
$M_{am}/(\gamma_{exh} - 1) = M_i - M_{bo} - M_{am}$
$M_{am} = (M_i - M_{bo})(\gamma_{exh} - 1)/\gamma_{exh}$,

so the energy efficiency, which is the ratio of the final vehicle kinetic energy to the total exhaust kinetic energy, is

$$\text{eff} = M_{bo}(\gamma_{end} - 1)\gamma_{exh}/[(M_i - M_{bo})(\gamma_{exh} - 1)]$$
$$= (\gamma_{end} - 1)\gamma_{exh}/[(M_i/M_{bo} - 1)(\gamma_{exh} - 1)]$$
$$= (\gamma_{end} - 1)\gamma_{exh}/[(r - 1)(\gamma_{exh} - 1)]$$

The relativistic rocket equation, in its "velocity-parameter" form, is

$$\Delta\theta = (V_{exh}/c) \ln r$$

and the definition of the increment in velocity parameter $\Delta\theta$ is

$$\tanh(\Delta\theta) = V_{end}/c$$

or $$\cosh(\Delta\theta) = \gamma_{end}$$

Note: $\cosh^{-1}(\gamma_{end}) = \ln[\gamma_{end} + \sqrt{\gamma_{end}^2 - 1}\ ]$

so $$r = \exp[\cosh^{-1}(\gamma_{end})/V_{exh}]$$

The expression for eff is evaluated using a Fortran computer program, OPTUEXH (see Addendum). The results here are for $\eta = 1$ (see Section II-B for arbitrary η) and DUMP = 0 (proton exhaust; see Section III for comparison with electron exhaust).

3. RESULTS

The results of the calculations of the maximum energy efficiency, the optimum V_{exh} and the minimum ratios of antimatter mass to burn-out mass and to initial mass, along with some side calculations of V_{end} and the optimum U_{exh}, are given in the table below for ascending values of the mission final proper velocity U_{end}. (Note: these calculations assume no energy losses in converting nuclear energy to exhaust kinetic energy.)

U_{end}	V_{end}	optV_{exh}	optU_{exh}	maxeff	U_{end}/optU_{exh}	minM_{am}/M_{bo}	minM_{am}/M_i	M_i/M_{bo}
------non-relativistic------				0.6476	1.59	---	---	4.92
0.2	0.1961	0.1239	0.1249	0.6472	1.60	0.0153	0.00308	4.97
0.5	0.4472	0.2910	0.3041	0.6454	1.64	0.0914	0.01749	5.23
1.0	0.7071	0.4924	0.5657	0.6405	1.77	0.3234	0.05398	5.99
2.0	0.8944	0.6912	0.9566	0.6301	2.09	0.9809	0.12151	8.07
3.0	0.9487	0.7772	1.235	0.6219	2.43	1.7385	0.16753	10.38
4.0	0.9701	0.8232	1.450	0.6155	2.76	2.5369	0.19917	12.74
5.0	0.9806	0.8516	1.625	0.6105	3.08	3.3570	0.22218	15.11

4. OBSERVATIONS

The minimized amount of antimatter, for mission proper velocities as high as 5 lt-yr/yr, is a small fraction, less than 25 percent, of the starship's initial mass.

The maximum energy efficiency decreases slowly as mission proper velocity is increased, but remains over 60 percent up to a mission proper velocity of 5 lt-yr/yr.

The ratio of the mission proper velocity to the optimum exhaust proper velocity increases fairly slowly at first from 1.59 at low velocities to almost double that at a mission proper velocity of 5 lt-yr/yr.

The implications of the values of the optimum exhaust velocity need to be examined in terms of their conversion to MeV for exhaust particles.

(added 5/2/96 to my personal file)
B. ARBITRARY CONVERSION EFFICIENCY

The conversion efficiency (η) is the fraction of the annihilation energy converted to exhaust kinetic energy.

1. ANALYSIS

The relativistic derivation of the maximum propulsive energy efficiency is–

final vehicle kinetic energy = $M_{bo} (\gamma_{end} - 1) c^2$
total exhaust kinetic energy = $M_p (\gamma_{exh} - 1) c^2 = \eta M_a c^2$
which gives $M_p = \eta M_a/(\gamma_{exh} - 1)$
but relativistically $M_p = M_i - M_{bo} - M_a$
$\eta M_a/(\gamma_{exh} - 1) = M_i - M_{bo} - M_a$
$M_a = (M_i - M_{bo})(\gamma_{exh} - 1)/(\gamma_{exh} - 1 + \eta)$

so the energy efficiency, which is the ratio of the final vehicle kinetic energy to the total exhaust kinetic energy, is

$$\begin{aligned} \text{eff} &= M_{bo} (\gamma_{end} - 1) c^2/(\eta M_a c^2) \\ &= M_{bo} (\gamma_{end} - 1) (\gamma_{exh} - 1 + \eta)/[\eta (M_i - M_{bo})(\gamma_{exh} - 1)] \\ &= (\gamma_{end} - 1)(\gamma_{exh} - 1 + \eta)/[\eta (M_i/M_{bo} - 1)(\gamma_{exh} - 1)] \\ &= (\gamma_{end} - 1)(\gamma_{exh} - 1 + \eta)/[\eta (r - 1)(\gamma_{exh} - 1)] \end{aligned}$$

The derivation of the relativistic rocket equation for an antimatter-powered rocket with the conversion efficiency η not equal to 1.0 is as follows–

The "velocity-parameter" equation of motion is

$$F = \gamma_{exh} V_{exh} dM_p/dt' = M c \, d\theta/dt' \quad ,$$

and the definition of the velocity parameter θ is (for V and U in lt-yr/yr; c = 1 lt-yr/yr)

$$\tanh \theta = V_{end}$$

or $\sinh \theta = U_{end}$.

The rate of change of ship mass equals the rate of ejection of propellant mass plus the rate of annihilation of mass, or

$$dM/dt' = dM_p/dt' + dM_a/dt' \quad ,$$

but from above

$$M_p = \eta M_a/(\gamma_{exh} - 1) \quad ,$$

or

$$M_a = (\gamma_{exh} - 1) M_p/\eta \quad ,$$

so

$$dM/dt' = dM_p/dt' + (\gamma_{exh} - 1)(dM_p/dt')/\eta$$

$$= (\gamma_{exh} - 1 + \eta)(dM_p/dt')/\eta$$

or

$$dM_p/dt' = \eta\ (dM/dt')/(\gamma_{exh} - 1 + \eta)$$

Substituting this in the velocity-parameter equation of motion gives

$$\gamma_{exh}\ V_{exh}\ \eta\ (dM/dt')/(\gamma_{exh} - 1 + \eta) = M\ c\ d\theta/dt'$$

or

$$d\theta = [\gamma_{exh}\ (V_{exh}/c)\ \eta/(\gamma_{exh} - 1 + \eta)](dM/M) \quad .$$

Integrating this from $\theta = 0$, when $M = M_i$, to $\theta = \Delta\theta$, when $M = M_{bo}$, (and measuring V_{exh} in lt-yr/yr; c = 1 lt-yr/yr) gives the relativistic rocket equation for η not equal to 1.0–

$$\Delta\theta = [\gamma_{exh}\ \eta/(\gamma_{exh} - 1 + \eta)]\ V_{exh}\ \ln(M_i/M_{bo}) \quad ,$$

whence, substituting $\sinh^{-1}(U_{end})$ for $\Delta\theta$, we get the expression for the mass ratio–

$$r = \exp[\sinh^{-1}(U_{end})\ (\gamma_{exh} - 1 + \eta)/(\gamma_{exh}\ \eta\ V_{exh})] \quad .$$

[Note: $\sinh^{-1}(U_{end}) = \ln(U_{end} + \sqrt{U_{end}^2 + 1}\ )$.]

With this relation we have all of the parameters to calculate

$$eff = (\gamma_{end} - 1)[\gamma_{exh} - 1 + \eta]/[\eta\ (r - 1)\ (\gamma_{exh} - 1)] \quad .$$

The expression for eff is evaluated using a Fortran computer program, OPTUEXH, a description and a copy of which are given as an Addendum. The calculations here were made with DUMP = 0 (proton exhaust; see Section III for calculations for electrons).

2. RESULTS

The results of the calculations of the optimum V_{exh}, the maximum energy efficiency and the minimum ratios of antimatter mass to burn-out mass and to initial mass are given in the table below for ascending values of the mission final proper velocity U_{end}, for conversion efficiencies from 1.0 to 0.5. Included in the table are values of V_{end}, to illustrate the degree of saturation of apparent velocity, and of the optimum U_{exh}, which is the actual parameter of optimization rather than V_{exh}.

The extreme U_{end} of 5 lt-yr/yr represents the final velocity reached at a continuous acceleration, a, of one g (1.0324 lt-yr/yr^2) over a distance of 3.97 lt-yr. (The acceleration distance $s = [\sqrt{1+U^2} - 1]/a$.) Only for destinations beyond about 8 lt-yr or accelerations greater than one g need one consider values of U_{end} greater than 5 lt-yr/yr.

The results for acceleration of protons are as follows:

(The tabulated quantities are–

U_{end} = final proper velocity, lt-yr/yr, of the starship at the end of the propulsive phase (i.e., at "burnout")

V_{end} = final apparent velocity, lt-yr/yr, ditto

[The apparent velocity V is given in terms of the proper velocity U by the relation

$V = U/\sqrt{1+U^2}$, (note: c = 1 lt-yr/yr),

and the inverse relation is

$U = V/\sqrt{1-V^2}$.]

$optV_{exh}$ = optimum apparent exhaust velocity, lt-yr/yr

$optU_{exh}$ = optimum proper exhaust velocity, lt-yr/yr

maxeff = maximum propulsive energy efficiency

$minM_{am}/M_{bo}$ = minimum ratio of initial mass of antimatter fuel to the ship's burnout mass

$minM_{am}/M_i$ = minimum ratio of initial mass of antimatter fuel to the ship's initial mass

M_i/M_{bo} = mass ratio = ratio of ship's initial mass to ship's burnout mass.)

Conversion Efficiency = 1.0

U_{end}	V_{end}	$optV_{exh}$	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1239	0.1249	0.6472	0.0153	0.00308	4.97
0.5	0.4472	0.2910	0.3041	0.6454	0.0914	0.01749	5.23
1.0	0.7071	0.4924	0.5657	0.6405	0.3234	0.05398	5.99
2.0	0.8944	0.6912	0.9566	0.6301	0.9809	0.12151	8.07
3.0	0.9487	0.7772	1.235	0.6219	1.7385	0.16753	10.38
4.0	0.9701	0.8232	1.450	0.6155	2.5369	0.19917	12.74
5.0	0.9806	0.8516	1.625	0.6105	3.3570	0.22218	15.11

Conversion Efficiency = 0.9

U_{end}	V_{end}	$optV_{exh}$	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1239	0.1248	0.6467	0.0170	0.00342	4.98
0.5	0.4472	0.2900	0.3030	0.6422	0.1021	0.01928	5.30
1.0	0.7071	0.4880	0.5591	0.6300	0.3653	0.05850	6.24
2.0	0.8944	0.6799	0.9271	0.6039	1.1370	0.12772	8.90
3.0	0.9487	0.7618	1.176	0.5828	2.0612	0.17253	11.95
4.0	0.9701	0.8056	1.360	0.5661	3.0647	0.20230	15.15
5.0	0.9806	0.8327	1.504	0.5526	4.1206	0.22341	18.44

Conversion Efficiency = 0.8

U_{end}	V_{end}	$optV_{exh}$	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1238	0.1247	0.6460	0.0192	0.00384	4.99
0.5	0.4472	0.2889	0.3017	0.6381	0.1156	0.02147	5.39
1.0	0.7071	0.4827	0.5512	0.6172	0.4195	0.06384	6.57
2.0	0.8944	0.6664	0.8939	0.5732	1.3477	0.13458	10.01
3.0	0.9487	0.7438	1.113	0.5383	2.5106	0.17781	14.12
4.0	0.9701	0.7850	1.267	0.5112	3.8183	0.20547	18.58
5.0	0.9806	0.8106	1.384	0.4896	5.2326	0.22461	23.30

Conversion Efficiency = 0.7

U_{end}	V_{end}	$optV_{exh}$	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1237	0.1247	0.6451	0.0219	0.00438	5.01
0.5	0.4472	0.2874	0.3001	0.6330	0.1332	0.02422	5.50
1.0	0.7071	0.4762	0.5416	0.6013	0.4921	0.07026	7.00
2.0	0.8944	0.6504	0.8563	0.5366	1.6453	0.14223	11.57
3.0	0.9487	0.7225	1.045	0.4871	3.1705	0.18342	17.29
4.0	0.9701	0.7607	1.172	0.4500	4.9576	0.20873	23.75
5.0	0.9806	0.7845	1.265	0.4210	6.9539	0.22578	30.80

Conversion Efficiency = 0.6

U_{end}	V_{end}	$optV_{exh}$	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1235	0.1245	0.6439	0.0256	0.00509	5.04
0.5	0.4472	0.2856	0.2980	0.6263	0.1570	0.02779	5.65
1.0	0.7071	0.4680	0.5295	0.5809	0.5942	0.07809	7.61
2.0	0.8944	0.6309	0.8131	0.4923	2.0923	0.15077	13.88
3.0	0.9487	0.6968	0.9716	0.4281	4.2095	0.18935	22.23
4.0	0.9701	0.7317	1.073	0.3817	6.8176	0.21204	32.15
5.0	0.9806	0.7533	1.145	0.3469	9.8464	0.22690	43.40

Conversion Efficiency = 0.5

U_{end}	V_{end}	$optV_{exh}$	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1233	0.1242	0.6422	0.0308	0.00608	5.07
0.5	0.4472	0.2830	0.2951	0.6171	0.1913	0.03259	5.87
1.0	0.7071	0.4572	0.5141	0.5538	0.7479	0.08791	8.51
2.0	0.8944	0.6064	0.7627	0.4378	2.8235	0.16037	17.61
3.0	0.9487	0.6653	0.8911	0.3595	6.0154	0.19561	30.75
4.0	0.9701	0.6961	0.9697	0.3061	10.2014	0.21537	47.37
5.0	0.9806	0.7152	1.023	0.2678	15.3048	0.22797	67.13

3. OBSERVATIONS

The minimized amount of antimatter remains a small fraction of the starship's initial mass, less than 25 percent for mission proper velocities as high as 5 lt-yr/yr, for all conversion efficiencies considered.

The maximum energy efficiency drops more rapidly with increasing mission proper velocity as conversion efficiency is decreased, and is barely above 25% for a conversion efficiency of 0.5 at a mission proper velocity of 5 lt-yr/yr.

III. PROTONS vs. ELECTRONS FOR RELATIVISTIC ELECTRIC THRUSTERS

A. QUESTIONS REGARDING RELATIVISTIC ELECTRIC THRUSTERS

(posted 1/25/97)

A critical component of the most interesting "near-term" interstellar propulsion concepts (e.g., fusion-powered rockets,* antimatter-powered rockets, beam-driven sails in the deceleration phase) is a relativistic electric thruster. This is not an "ion rocket," which typically has ion energies in the multi-kilovolt range and ion currents in milliamps. It is a high-energy particle accelerator with an exhaust-particle energy in the range of multi-megavolts to gigavolts (see the 1,060 MeV proton-energy requirement for a thruster to decelerate a beam-driven sail, in Section V-C) and particle currents in the range of kiloamps to megamps (up to 2,940 megawatts of beam power per kg of thrust, see Sections V-C and V-D). [While 2,940 Mw of photons (either propellant photons or collected emitter-beam photons) or 2,940 Mw of 1,000-MeV, say, electrons give 1 kg of thrust, 2,940 Mw of 1,000-MeV protons give 1.6961 kg of thrust. The number of kg of thrust for 2,940 Mw of proton beam power is given by the value of RBE in Section V-C. ...posted 3/16/01 in review]

I think it would be useful for us to start off by examining the characteristics of these particle accelerators to help select the most desirable configuration. In this vein, I would like to suggest a sample list of questions regarding these thrusters, to draw out any further questions you all might think of. Phrasing the question right is a significant part of solving the problem.

The sample tentative questions are as follows:

1. What is the best exhaust particle? Electrons, protons, alphas, etc.? What is the best parameter to compare them by?
   a. What is the relative thrust/amp for the options, as a function of their energy in MeV?
   b. What is their corresponding exhaust velocity?
   c. What is their "I_{sp}" (thrust per unit rest-mass flow rate)?

2. Is there an electron energy that gives the same performance as protons? (from the so-called "mass amplification" with increasing velocity)

3. Can we estimate relative thruster weight per unit thrust? Thruster size? (to compare the different options)

4. What disadvantages/advantages accompany motion of electric charges?

---

* On 1/30/97 Timothy wrote–

> "Actually a *relativistic* thruster is probably not useful for a fusion-powered design, since such designs in general have an optimum for rather 'slow' exhaust velocities (less than 0.l c).
>
> "So only beam-powered or antimatter-powered ships are candidates for relativistic thrusters."

a. What is the effect on performance (e.g., exhaust velocity) of charging of the ship due to incomplete charge neutralization?
b. What is the effect on performance of charge neutralization obtained by dumping oppositely charged particles (at "zero" velocity, say)?
c. Will the current of the exhaust beam generate a magnetic field strong enough to deflect interstellar hydrogen atoms (ionized to protons by a stripping foil out in front of the ship) away from the ship, to save shielding?
d. Could the electric charge from incomplete neutralization create an electric field, around a sharp point out in front of the ship, strong enough to deflect interstellar protons as in c, above?
e. Could electrodes on the sides of the particle beam at the exit end of the accelerator be used to steer the "jet" (and hence the ship)?
f. Is there a possibility of entrainment of interstellar protons by the particle beam to cause jet augmentation?
g. What might be the damage caused by impingement of the very-high-powered particle beam on another object in space?

This list is not meant to be exhaustive nor definitive, but to stimulate further thought and discussion (not all issues are addressed in the following).

(posted 2/4/97)
(A news item in Scientific American, April 1987, p. 66, describes plasma wake-field acceleration of electrons with a projected gradient of one billion volts per meter. The item, however, did not include information that would support an expectation that that technology could provide the kinds of current or the efficiencies that we must have for our thrusters.)

Some selected quotes from Ken and Timothy that set the stage for the question I want to address in this Section are the following:

On 1/27/97 Ken wrote–
>1) and 2)...And everything points to low mass particles being the best.

>Assuming that the final energy of the particles will be large compared to the rest >mass,...the rest mass becomes irrelevant to the momentum...

>The Big Problem, of course, is keeping the ship neutral. Assuming we don't have >positrons handy...we need one proton per electron, which will severely hurt the I_{sp}.

On 1/30/97 Timothy wrote (quoting me)–
>>1. What is the best exhaust particle? Electrons, protons, alphas, etc.? What is the best >>parameter to compare them by?

>Actually to determine an optimum we should first decide what we want to optimize. ... >High velocities have a low momentum:energy ratio, but of course need a lot less mass.

>So you always have to weigh between how much mass and how much energy. Also one >would want to use most of the repulsion mass that is taken with the starship, this almost >certainly means that one needs to use ions (thus not electrons). For the highest exhaust >velocity, one should take the particle with the highest charge:mass ratio, this would have >been an electron, so the next best thing would be a proton...

On 1/30/97 Timothy wrote (quoting Ken)–
>>3) In terms of size... Would there be an optimal length? I would guess no: you want >>the device as long as possible...
>>Doubling the length will not double the mass of your entire ship, but it will double the >>amount of thrust you can get!

>Previous calculations have shown that optimal exhaust velocity depends (among other >things) on the final velocity of the starship. Just creating the highest exhaust velocity is >therefore not the main goal.

So, from these points by Ken and Timothy, I have been able to distill an important question that I can start to address quantitatively:

Q: What is the optimum (minimum-antimatter) performance of an antimatter-powered starship with its exhaust composed of accelerated protons (with electrons dumped for charge neutralization), in comparison with that of one with its exhaust composed of accelerated electrons (with protons dumped at negligible velocity for charge neutralization)?

B. ACCELERATED PROTONS

The performance of an antimatter-powered starship with a proton exhaust velocity that is optimized to require a minimum mass of antimatter fuel was derived in Section II.

The circumstance of minimum antimatter fuel is obtained by finding the exhaust velocity, for a given final vehicle velocity, that maximizes the propulsive energy efficiency, i.e., maximizes the conversion of exhaust kinetic energy to final vehicle kinetic energy. (The overall efficiency of conversion of antimatter energy to final vehicle kinetic energy is the product of this maximum propulsion energy efficiency and η, the efficiency of conversion of antimatter energy to exhaust kinetic energy.)

Section II includes a derivation of the calculational procedure by which the results were obtained.

The Section II results for acceleration of protons with $\eta = 1$ are repeated below (see Section II-B for results with $\eta < 1$):
(The tabulated quantities are–
U_{end} = final proper velocity, lt-yr/yr, of the starship at the end of the propulsive phase (i.e., at "burnout")

V_{end} = final apparent velocity, lt-yr/yr, ditto
[The apparent velocity V is given in terms of the proper velocity U by the relation

$V = U/\sqrt{1+U^2}$, (note: c = 1 lt-yr/yr),

and the inverse relation is

$U = V/\sqrt{1-V^2}$.]

$optV_{exh}$ = optimum apparent exhaust velocity, lt-yr/yr
$optU_{exh}$ = optimum proper exhaust velocity, lt-yr/yr
maxeff = maximum propulsive energy efficiency
$minM_{am}/M_{bo}$ = minimum ratio of initial mass of antimatter fuel to the ship's burnout mass
$minM_{am}/M_i$ = minimum ratio of initial mass of antimatter fuel to the ship's initial mass
M_i/M_{bo} = mass ratio = ratio of ship's initial mass to ship's burnout mass.)

Antimatter conversion efficiency = 1.0

U_{end}	V_{end}	$optV_{exh}$	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
------non-relativistic------				0.6476	---	---	4.92
0.2	0.1961	0.1239	0.1249	0.6472	0.0153	0.00308	4.97
0.5	0.4472	0.2910	0.3041	0.6454	0.0914	0.01749	5.23
1.0	0.7071	0.4924	0.5657	0.6405	0.3234	0.05398	5.99
2.0	0.8944	0.6912	0.9566	0.6301	0.9809	0.12151	8.07
3.0	0.9487	0.7772	1.235	0.6219	1.7385	0.16753	10.38
4.0	0.9701	0.8232	1.450	0.6155	2.5369	0.19917	12.74
5.0	0.9806	0.8516	1.625	0.6105	3.3570	0.22218	15.11

In the accelerated-protons case, we have an optimum exhaust velocity and therefore an optimum accelerator length, as Timothy states.

[(posted 2/20/97 to Timothy)
>Still other ions may work as well...

I believe the results for protons apply equally well to any ion whose mass makes the mass of an electron dumped for charge neutralization negligible.]
(Of course, alphas with the same velocity as protons have four times the energy.)

C. ACCELERATED ELECTRONS

The calculational procedure described in Section II-B is modified, to incorporate dumping of mass at negligible velocity to bring about charge neutralization, by changing the basic mass-flow equation (using the notation of Section II-B) from–

$dM/dt' = dM_p/dt' + (\gamma_{exh} - 1)\ (dM_p/dt')/\eta$

to–

$dM/dt' = dM_p/dt' + (\gamma_{exh} - 1)\ (dM_p/dt')/\eta + DUMP\ (dM_p/dt')$

(where DUMP = 1/1836 ≈ 0 for protons and DUMP = 1836 for electrons), and replacing the expression–

$(\gamma_{exh} - 1 + \eta)/\eta$ [which is equal to $(\gamma_{exh} - 1)/\eta + 1$],

where it appears thereafter, with–

$(\gamma_{exh} - 1)/\eta + 1 + DUMP$.

The comparable results of the calculations for accelerated electrons (with dumping of one proton at negligible velocity for each electron for charge neutralization: "DUMP = 1836.") are as follows with the same nomenclature as above (Note: in all cases the value of $optV_{exh}$ is now 1.0000):

(For electrons, there is no maximum efficiency as a function of U_{exh} for finite values of U_{exh}; the efficiency increases monotonically as U_{exh} is increased. The maximum is replaced by an asymptote at infinite U_{exh}. The value of "maxeff" tabulated below is that efficiency when the increase in efficiency is 0.001 percent for an increase in U_{exh} of 1 percent; the tabulated value is within about 0.1 percent of the asymptote.)

Conversion Efficiency = 1.0

U_{end}	V_{end}	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	$0.1838x10^6$	0.0900	0.1100	0.09001	1.22
0.5	0.4472	$0.4778x10^6$	0.1908	0.3093	0.19082	1.62
1.0	0.7071	$0.9307x10^6$	0.2926	0.7078	0.29268	2.42
2.0	0.8944	$0.1625x10^7$	0.3816	1.6197	0.38173	4.24
3.0	0.9487	$0.2168x10^7$	0.4184	2.5837	0.41863	6.17
4.0	0.9701	$0.2568x10^7$	0.4380	3.5651	0.43823	8.14
5.0	0.9806	$0.2894x10^7$	0.4500	4.5540	0.45028	10.11

Conversion Efficiency = 0.9

U_{end}	V_{end}	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	$0.1913x10^6$	0.0890	0.1236	0.09896	1.25
0.5	0.4472	$0.4778x10^6$	0.1853	0.3538	0.20690	1.71
1.0	0.7071	$0.9400x10^6$	0.2765	0.8321	0.31199	2.67
2.0	0.8944	$0.1674x10^7$	0.3453	1.9885	0.39922	4.98
3.0	0.9487	$0.2212x10^7$	0.3669	3.2743	0.43348	7.55
4.0	0.9701	$0.2646x10^7$	0.3747	4.6305	0.45102	10.27
5.0	0.9806	$0.2981x10^7$	0.3774	6.0348	0.46151	13.08

Conversion Efficiency = 0.8

U_{end}	V_{end}	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	$0.1875x10^6$	0.0877	0.1411	0.10986	1.28
0.5	0.4472	$0.4826x10^6$	0.1787	0.4129	0.22582	1.83
1.0	0.7071	$0.9589x10^6$	0.2574	1.0057	0.33362	3.01
2.0	0.8944	$0.1708x10^7$	0.3040	2.5411	0.41749	6.09
3.0	0.9487	$0.2234x10^7$	0.3100	4.3589	0.44828	9.72

4.0	0.9701	0.2646×10^7	0.3068	6.3633	0.46334	13.73
5.0	0.9806	0.3011×10^7	0.3010	8.5101	0.47204	18.03

Conversion Efficiency = 0.7

U_{end}	V_{end}	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1932×10^6	0.0861	0.1643	0.12345	1.33
0.5	0.4472	0.4923×10^6	0.1704	0.4948	0.24837	1.99
1.0	0.7071	0.9782×10^6	0.2344	1.2624	0.35781	3.53
2.0	0.8944	0.1759×10^7	0.2570	3.4356	0.43620	7.88
3.0	0.9487	0.2348×10^7	0.2482	6.2231	0.46258	13.45
4.0	0.9701	0.2726×10^7	0.2354	9.4768	0.47473	19.96
5.0	0.9806	0.3102×10^7	0.2232	13.1168	0.48145	27.24

Conversion Efficiency = 0.6

U_{end}	V_{end}	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1932×10^6	0.0840	0.1965	0.14083	1.40
0.5	0.4472	0.4874×10^6	0.1598	0.6157	0.27557	2.23
1.0	0.7071	0.9880×10^6	0.2062	1.6740	0.38468	4.35
2.0	0.8944	0.1813×10^7	0.2040	5.0501	0.45470	11.11
3.0	0.9487	0.2443×10^7	0.1826	9.8660	0.47568	20.74
4.0	0.9701	0.2837×10^7	0.1634	15.9285	0.48460	32.87
5.0	0.9806	0.3196×10^7	0.1478	23.1156	0.48925	47.25

Conversion Efficiency = 0.5

U_{end}	V_{end}	$optU_{exh}$	maxeff	$minM_{am}/M_{bo}$	$minM_{am}/M_i$	M_i/M_{bo}
0.2	0.1961	0.1913×10^6	0.0811	0.2442	0.16381	1.49
0.5	0.4472	0.5174×10^6	0.1458	0.8098	0.30879	2.62
1.0	0.7071	0.1038×10^7	0.1714	2.4166	0.41398	5.84
2.0	0.8944	0.1924×10^7	0.1458	8.4804	0.47195	17.97
3.0	0.9487	0.2492×10^7	0.1168	18.5055	0.48667	38.02
4.0	0.9701	0.2981×10^7	0.0960	32.5250	0.49228	66.07
5.0	0.9806	0.3427×10^7	0.0811	50.5447	0.49497	102.12

(Note: This problem would have been almost hopelessly difficult if the parameter of optimization had been the conventional *apparent* exhaust velocity rather than the *proper* exhaust velocity; for the last line, the apparent exhaust velocity is 0.99999999999994 lt-yr/yr for the stated proper exhaust velocity.)

[(posted 2/20/97 to Timothy, regarding the $\eta = 1$ results)
>I'm amazed that dumping so much mass can still give efficiencies over 25%.

Remember, when the γ is very high (like 10^6 or more), the effective mass of the accelerated electron makes the *proton* mass negligible. The results merely reflect the effect of the protons in removing the efficiency peak at "intermediate" U_{exh}. For a proton

exhaust, I seem to recall from earlier calculations that the efficiency is also asymptotic to about 45 percent at infinite U_{exh}, but it approaches the asymptote from above.]

In the accelerated-electrons case, the kinetic energy efficiency stays virtually constant with increasing exhaust velocity above the tabulated "optimum" value (cf. Note below). In this case Ken is right (thrust increases directly with exhaust velocity without limit; "you want the device as long as possible").

[(posted to Timothy 2/20/97)
>Since you propose that infinite exhaust speeds are the best, isn't Steve right saying that >you could better use photons as propulsion?

I quote p. 20 of the Appendix: "Note: a 'photon rocket' with the same light-beam power would have the same thrust. The choice between electrons and photons depends on the mass of the beam producer and power supply."]

D. COMPARISON

Timothy observed (above), "you always have to weigh between how much mass and how much energy."

A succinct comparison between protons and electrons can be made with a table of the principal mass-related and energy-related properties of starships that would make use of the two choices of exhaust particles.

The particle-accelerator energy for a specified U_{exh} is given by the relation

$$\text{particle kinetic energy (MeV)} = mc^2 \ [\sqrt{1 + U_{exh}{}^2} - 1] \ ,$$

where mc^2 is 938.3 MeV for protons and 0.511 MeV for electrons.

For the protons' $optU_{exh}$ of 1.625 lt-yr/yr for the U_{end} of 5.0 lt-yr/yr (achieved at a continuous acceleration of 1 g over a distance of 3.97 lt-yr), the proton-accelerator energy is about 850 MeV.

For the electrons' "$optU_{exh}$" of 0.2894×10^7 lt-yr/yr for the U_{end} of 5.0 lt-yr/yr (for η = 1.0), the electron-accelerator energy is about 1,480 GeV.
(Note: This energy, on a flat asymptotic part of the performance curve, could probably be reduced several orders of magnitude with small effect on the required mass ratio. It is not until the electron energy is reduced below 3.11 MeV that the V_{exh} falls below 0.99 lt-yr/yr. A real optimization would balance the increase in fuel to propel a heavier electron accelerator against the increase in fuel associated with a reduced V_{exh}.)

The thrust T is given by the relation (see Appendix, pp. 19-20)

$$T = iV \ \sqrt{1 + (2mc^2 / MeV)} \ \text{(1 kgf/2,940 Mw)} \ ,$$

where mc^2 is as above, i is current in amps, V is volts and MeV is the accelerator energy; (1 amp) x (1 volt) is 1 w.

The values extracted from the $U_{end} = 5.0$ lines in the $\eta = 1.0$ tables above or calculated from the above relations are as follows:

Property	Proton exhaust	Electron exhaust
Mass ratio	15.11	10.11
Maxeff	0.610	0.450
$MinM_{am}/M_{bo}$	3.357	4.554
$MinM_{am}/M_i$	0.2222	0.4503
Accelerator Energy (MeV)	850.	1,480,000. (see Note above)
Thrust/amp (kgf)	0.520	500.

[(posted 2/20/97 to Timothy)
I suspect that, if the electron energy were reduced by a factor of 1000 [making the thrust/amp values essentially the same], there would be only a small decrease in kinetic-energy efficiency, and then the difference between the two would reduce just to the difference in efficiencies, which I agree outweighs the difference in mass. (Of course, an electron accelerator will be far lighter than a proton accelerator of the same energy.)]

"You pays your money and you takes your choice."

IV. THE SIZE OF THE PROBLEM

(posted 8/4/96; this is an expansion of a Timothy post of 11/4/94)
INTRODUCTION

A challenge has been identified to convey to the SD Group the size of the problem of providing the energy required for interstellar flight.

Let us consider an example mission to deliver 100 tonnes to a distance of 8 light-years at a continuous acceleration/deceleration of 1 g, a mission for which we have some numbers.

CALCULATIONS

A. ENERGY CONTENT of ANTIMATTER

The specific energy of annihilation (also creation) of 1 kg of matter-plus-antimatter (m+am) is

mc^2 = 1 kg x $(2.99792458 \times 10^8$ m/sec$)^2$
= 8.98755×10^{16} kg m^2/sec^2
.....(1 kg m^2/sec^2 = 1 joule = 10^{-6} Mw-sec)
= 8.98755×10^{10} Mw-sec
.....(1 yr = 3.15576×10^7 sec)
= 2,847.98 Mw-yr (per kg of m+am)

B. ELECTRIC ENERGY PRODUCTION in the US

The 1994 Grolier Encyclopedia says, under *power, generation and transmission of*: "In 1987, production of electric energy by utilities in the United States totaled 2,570 billion kilowatt-hours."

$2{,}570 \times 10^9$ kw-hr x 3600 sec/hr / (1000 kw/Mw x 3.15576×10^7 sec/yr)
= 293,178 Mw-yr

Let us call 293,178 Mw-yr one "USE." Then

1 USE = 293,178 Mw-yr / 2,847.98 Mw-yr/kg m+am
= 102.9425 kg m+am
= 51.471 kg am

C. ANTIMATTER MASS REQUIREMENT

In Section II-A, I gave the calculated values of the ratios of the minimum antimatter mass to the burnout mass ($minM_{am}/M_{bo}$) and the minimum antimatter mass to the initial mass ($minM_{am}/M_i$) required for various values of the final proper velocity (U_{end}) up to a U_{end} of 5 lt-yr/yr. The assumptions in the calculations were–

1. 100 percent conversion of annihilation energy to exhaust kinetic energy (unrealistic, but adopted to simplify the initial calculations), and
2. The exhaust velocity was that giving the maximum conversion of exhaust kinetic energy to vehicle kinetic energy, i.e., that requiring the minimum mass of matter+antimatter, for each final velocity.

For acceleration to a U_{end} of 5, the value of $minM_{am}/M_{bo}$ given in Section II-A is 3.3570 and the value given for $minM_{am}/M_i$ is 0.22218. With these numbers, the mass ratio for the acceleration phase given by $minM_{am}/M_{bo}$ divided by $minM_{am}/M_i$ is 15.109, including the exhaust mass and the matter annihilated with the antimatter. The overall mass ratio for acceleration plus deceleration is $(15.109)^2$, or 228.28. The ratio of antimatter mass to burnout mass is 23.79 percent (3.3570/14.109) of the "fuel ratio" (the mass ratio minus one). The overall ratio of antimatter mass to burnout mass is therefore 54.32.

A U_{end} of 5 lt-yr/yr is achieved at an acceleration of 1 g over a distance of 3.97 lt-yr (see Section II-B-2). So, a peak proper velocity of 5 lt-yr/yr is reached halfway to a destination about 8 lt-yr away, and the antimatter requirement is 54.32 kg of antimatter for each kg of mass delivered to that destination.

With one more assumption–

3. 100 percent efficiency of conversion of electric energy to antimatter (again unreal but simplifying),

the minimum energy requirement to create the antimatter to deliver 1 kg to a distance of 8 lt-yr at 1-g continuous acceleration/deceleration becomes

54.32 kg am / 51.471 kg am/USE = 1.0553 USE.

So the energy requirement to deliver an example final mass of 100 tonnes (10^5 kg) to an example distance of 8 lt-yr at an acceleration/deceleration of 1 g is 1.055×10^5 USE, or 105,500 years of the rate of electric energy production in the entire US in the year 1987, if the efficiency of conversion of electric energy to antimatter is 100 percent.

IMPLICATIONS

Similar considerations could be applied to relate to the USE the fusion-powered-rocket energy requirement, the sail/beam energy requirement, etc. All will certainly be about as much beyond current power-generation capabilities as antimatter creation is. Human interstellar flight in a human lifetime is far beyond current engineering/economics. Orders of magnitude reduction in payload and increase in transit time will be required to reduce the energy problem to a "manageable" size, to only a few USEs, say.

What to do about the energy problem for human starflight?

A few plausible ways around the problem (requiring extensions of physics, however) come to mind:

1. Find some process to make antimatter which does not require creation energy, such as changing matter to antimatter through some kind of quantum manipulation (transmutation). On 3/22/96 Kelly wrote (to Lee), paraphrased, "So physicists are talking about the possibility of rotating the quantum particles to convert a particle of matter to antimatter." (To which Lee added, "Assuming it doesn't require more energy to rotate the particle than the energy it produces..." in a matter-antimatter reaction.)

2. Discover some ultra-cheap source of energy (cold fusion?). [Note that 1 cubic mile of (sea) water contains enough deuterium to provide about 5,400 USEs from the fusion reaction(s) $D + D + D \longrightarrow He^4 + n + p + 21.6$ MeV (for which f = 261).]

3. Tunnel through space ("warp drive"?).

The group might think of others.

On 8/5 Kelly wrote–
>A few come to mind.
>
>1) Don't use antimatter.

And use at least 250 times the mass of energy fuel?

On 8/12 Kelly wrote–
>1) Hell yes, I'd rather have hundreds of times the fuel weight! Obviously due to the >weight of the fuel you'd need more than 250 times as much, but it would be a lot easier >to carry!

Using antimatter fuel, the example mission to 8 lt-yr at 1-g continuous acceleration/ deceleration is calculated (see Section IV-C) to require a mass ratio of at least 15.11 for the acceleration phase alone. Using deuterium fuel in the above fusion reaction (with the mass/energy ratio f = 261) for the same mission, the calculated required mass ratio for the

acceleration phase is at least 3.84 x 10^{12} kg D_2 for each kg delivered to the peak velocity half way to the destination. (Note: This mass ratio is for an "optimum" exhaust velocity of 0.09580 c. One of the reaction products is a neutron, which can't be used as reaction mass.) With 1.44 x 10^8 kg D_2 per cubic mile of ocean, this mass ratio says you have to extract all the deuterium from more than 26,000 cubic miles of ocean to provide the fusion fuel to accelerate 1 kg at 1 g over 4 lt-yr, half way to the destination.

On 8/5 Kelly wrote–
>2) Don't try for a constant 1-g acceleration for the full duration of the flight.

While I believe Kelly may have had a coast phase in mind, his comment suggested to me trying values of constant acceleration/deceleration less than 1 g. With a slight modification to the calculation in Section II, I get the following reductions in energy requirement, in USE per kilogram of burnout mass, as the accel/decel value is decreased from 1 g (included in the table are the peak proper velocity U_{end} in lt-yr/yr, the trip time in yr and the mass ratio for the acceleration phase alone):

Accel/decel (g)	U_{end} (lt-yr/yr)	Energy/M_{bo} (USE/kg)	trip time (yr)	mass ratio(accel phase)
Distance = 8 (actually 7.941) lt-yr				
1.0	5.000	1.0553	4.480	15.11
Distance = 4.35 lt-yr (α Centauri)				
1.0	3.088	0.4067	3.576	10.58
0.9	2.851	0.3476	3.810	10.03
0.8	2.611	0.2925	4.087	9.47
0.7	2.369	0.2415	4.421	8.91
0.6	2.124	0.1946	4.835	8.35
0.5	1.872	0.1518	5.366	7.79
0.4	1.613	0.1132	6.083	7.22
0.3633	1.516	0.1000	6.417	7.01
0.3	1.342	0.0786	7.128	6.65
0.2	1.049	0.0483	8.867	6.08
0.1	0.707	0.0220	12.751	5.50
0.0476	0.475	0.0100	18.653	5.19

(The Fortran program TRIP that did the calculations is included as an Addendum.)

A plot of energy vs trip time shows an elbow at about 0.5 or 0.4 g, where additional expenditure in energy (for higher acceleration) starts to reach diminishing returns in reduction in trip time. The point at 0.3633 g for the α Centauri trip represents a reduction in USE/kg by about an order of magnitude from the example 8-lt-yr, 1-g trip. The point at 0.0476 g (for α Centauri) represents two orders of magnitude reduction, but at a significant increase in trip time (to 18.653 yr).

For the fusion-powered rocket with f = 261 considered above in response to the suggestion "Don't use antimatter," there is a dramatic reduction in required mass ratio as the g-level is reduced (and the trip time is increased). The following table shows that the required mass ratio is reduced by about 9 orders of magnitude while the trip time is increased by about a factor of five as the g-level is reduced from 1.0 to 0.05:

8 lt-yr trip:

accel/decel (g)	U_{end} (lt-yr/yr)	trip time (yr)	mass ratio (accel phase)
1.0	5.000	4.480	3.842×10^{12}
0.5	2.881	6.897	4.869×10^{9}
0.2	1.520	11.692	3.659×10^{6}
0.1	0.994	16.991	5.929×10^{4}
0.05	0.672	24.401	2.675×10^{3}
0.0132	0.333	48.064	6.057×10

For the last line, the mass ratio for the full trip would be only about 60^2 or about 3,600 kg D_2 for each kg of final mass, so one would have to process only about 2.5×10^{-5} cubic miles of water for each kg delivered to 8 lt-yr in a continuous-g trip time of about 48 yr (cut in half to about 24 yr with l-g accel/coast/l-g decel using the same energy; see below). To deliver Kelly's 500,000-ton-dry-weight Explorer-class starship on this mission would require extracting all the deuterium from about 12,500 cubic miles of water.

This may be a more "down-to-Earth" measure of the size of the problem. Now we're addressing some of the "difficulty in manufacturing."

On 8/12 Kelly wrote–
>You'd probably get shorter trip times if you had used the same energy in higher boosts at >the start and end of the trips, with a coast phase in the middle. Same energy >consumption, but higher average speed.

The following table shows the comparative trip times (to α Centauri: distance = 4.35 lt-yr) for continuous accel/decel at g levels below 1, with trip times for l-g accel/coast/l-g decel with the same peak velocity (same energy requirement). The values of peak velocity, Energy/M_{bo} and mass ratio (accel alone) for each entry are given in a previous table.

Continuous accel/decel		l-g accel/decel w/coast using same energy	
Accel/decel (g)	trip time (yr)	trip time (yr)	coast time (yr)
1.0	3.576	3.576	0.000
0.9	3.810	3.582	0.152
0.8	4.087	3.603	0.334
0.7	4.421	3.646	0.551
0.6	4.835	3.720	0.819
0.5	5.366	3.845	1.162
0.4	6.083	4.051	1.619
0.3	7.128	4.407	2.269
0.2	8.867	5.091	3.317
0.1	12.751	6.813	5.537

For acceleration periods that become a smaller fraction of the trip time, the average speed tends toward twice the average speed for continuous accel/decel, leading to cutting the trip time just in half.

Note: For a reduction in energy requirement by a factor of ten from the 1-g-all-the-way α Centauri trip (an accel/decel g of about 0.17, from the earlier reduced-acceleration table), the trip time for 1 g with coast for the same energy is increased by less than a factor of 2.
A factor of ten reduction in energy requirement (for antimatter fuel) for less than a factor of two increase in time says the coast phase is worth considering. On the down side, however, there are the requirements for heavier thruster (and power-system) weight for 1 g vs. reduced g, and increased complexity for artificial-gravity provisions during the coast time (the last column).

On 8/12 Kelly wrote–
>Thought: What is the relative weight of an antimatter tank to the weight of the >antimatter? Would its weight added to the ship start outweighing the advantages of the >lighter fuel?

If the deuterium for the fusion engine is carried in a tank made of an alloy of lithium and aluminum, the anti-hydrogen could be carried in a tank made of an alloy of anti-lithium and anti-aluminum (;-)), with a mass fraction similar to that of the deuterium tank.
["mass fraction" = (mass of contents)/(sum of masses of tank and contents)]
And when all the anti-hydrogen is annihilated with hydrogen, the anti-lithium/anti-aluminum tank can be used as additional antimatter fuel annihilating with lithium and aluminum.

ANTIMATTER ANNIHILATION PRODUCTS

(posted 2/6/97)
On 2/4 Steve wrote (quoting me)–
>>Antimatter conversion efficiency = 1.0.
>
>That's a common misconception. Unfortunately the annihilation products of matter-
>antimatter reactions aren't all photons, and it's ... hard even to get the matter and
>antimatter to combine completely under any realistic circumstances. My understanding
>(bolstered by some half-remembered statements by Robert Forward) is that the practical
>conversion efficiency is more like 0.8.

I have before me Robert Forward's report "Antiproton Annihilation Propulsion," AFRPL TR-85-034, September 1985. It may have been superseded by more recent information, but should be a good reference point from which to judge progress. He said at that time (on p. 109, et seqq):

"When an antiproton annihilates with a proton, the predominant reaction products (98%) are pions. A recent survey of the literature [1984 reference] *found that on the average there are 3.0 charged pions, 1.5 neutral pions, 0.05 charged kaons, 0.03 neutral kaons, and 0.02 prompt gamma rays. ... The neutral pions have a lifetime of only 90 attoseconds and almost immediately convert into two high-energy gamma rays."*

The charged pions are stopped by matter and deposit their energy (about two-thirds of the total) locally as heat. The energy of the gamma rays is harder to capture.

Forward continued:

"In many applications of the use of antiprotons for energy storage and propulsion, consideration is being given to annihilation of the antiprotons with heavier nuclei than protons. Since a neutron has the same baryon number as a proton and a free neutron will spontaneously decay into a proton, a neutron can be considered as an 'excited state' of a proton. Thus, antiprotons, will *annihilate with a neutron as well as a proton inside a heavy nucleus. Since the neutron has a neutral charge and charge must be conserved in the annihilation process, the reaction products from the annihilation of an antiproton in a heavy nucleus will produce different numbers of the various types of charged and uncharged pion and kaon particles.*

"Annihilation inside a heavy nucleus has the potential for increasing the efficiency of an antiproton annihilation propulsion system, since the neutral pions are absorbed in the nucleus instead of decaying into gamma rays. The annihilation reaction will 'heat up' the nucleus as well as cause spallation fission of charged nuclear fragments."

...

"An ideal reaction would be one where the energy from the antiproton causes the nucleus to break up into doubly charged alpha particles..."

Beryllium 9? Carbon 13? ... (We could accelerate alphas.)

And after we convert antimatter annihilation energy to usable thermal (or whatever) energy, we still have to convert the usable energy to exhaust kinetic energy (via some kind of engine/electrical generator powering an accelerator). The "conversion efficiency" that I used in the calculations reported in Sections II-B and III-C is the overall efficiency of conversion of antimatter energy to exhaust kinetic energy, i.e., the product of the conversion efficiency of antimatter to usable energy *and* the conversion efficiency from usable energy to exhaust kinetic energy. It could therefore be even lower than the 0.8 that Steve quotes.

It is to be hoped that the ultimate conversion from antimatter energy to exhaust kinetic energy will be greater than 0.5 (see Sections II-B-2 and III-C). The antimatter-mass requirement grows considerably as the overall conversion efficiency decreases.

V. MOTION OF BEAM-DRIVEN SAIL

A. POWER REQUIREMENTS FOR CONSTANT ACCELERATION OF SAIL

On 8/20/96 Steve wrote–
>While I haven't had time to do a more complete write-up, I thought I would also >mention an interesting corollary to Rex's energy-requirements analysis, that of beaming >power to accelerate a relativistic spacecraft at a *constant acceleration*. (Emphasis added) >Not only is a large amount of power required, but the beaming equipment must be >capable of (typically) output at a rate that can be over two orders of magnitude greater >than is needed to accelerate the spacecraft at the start of the trip.
(See also note by Kevin on 2/6/95 at 16:58 CST.)

>I'm going to present some of the math without proof or demonstration at this time, but >I'm sure it will be interesting fodder for discussion because it shows another facet of >difficulty to the problem of beaming power.

>I've recently been working on the physics of light signals between a "stationary" object >and an object undergoing relativistic acceleration relative to it. Consider an object >undergoing uniform acceleration relative to itself; its frame position at its proper time t' >is:
> $[t, x] = (1/a) [\sinh(a t'), (\cosh(a t') - 1)]$

>At time t' = 0, its position is [0, 0] (note that for simplicity I am using geometrized >units where c = 1 and acceleration has units 1/sec (acceleration is fraction of c per unit >time)). Consider an "antenna" at that position beaming power to accelerate the object, >so that at its time t = 0 it coincides with the accelerated object at its proper time t' = 0. If >energy (electromagnetic radiation, e.g., light) from the beamer is emitted at time t, then >the time t' at which the accelerated object receives the energy is:
> $t' = -(\ln(1 - a t))/a$.

>Note that this is an asymptotic relationship -- as the frame time t of the beamer >approaches 1/a, the object proper time t' approaches infinity. This consequently means >that the beamer must send energy for any possible trip within a time 1/a, no matter how >far the accelerated object goes, and that the rate at which power is sent increases >asymptotically to infinity as t approaches 1/a.
(See also notes by Kevin on 2/12/95 and by Steve on 8/15/95.)

On 9/4/96 Steve wrote–
>For example, if you are beaming power to accelerate a ship at 1 g, you have just short of >a year after its departure to beam power to it, no matter how far away you are sending it, >and you must beam increasing amounts of power towards the end of that time to sustain >the *constant* 1-g ship-frame acceleration. (Emphasis added)

>I presented Rex with the derivation that led me to that conclusion. Unfortunately, my >derivation was based on the solution to a substantially more complicated problem (where >does an observer at an arbitrary spacetime location see an accelerated object travelling in >an arbitrary direction?), which we agreed is not particularly accessible to the uninitiated. >Rex came up with a simpler derivation specific to this problem which is much easier to >understand, and which I will quote below.

"Actually, the description of the space-time diagram in your 8/26/96 note [reproduced below] was more effective in conveying the relation to my intuition. Having gone through that thought process, I can now rewrite the derivation more succinctly:

"The distance, x, of the sail after accelerating at a level, a, for a proper time, t', is

$$x = [\cosh(a\,t') - 1]/a \quad .$$

"The apparent (Earth) time, t, at which the sail reaches x is

$$t = [\sinh(a\,t')]/a \quad .$$

"The time, Δt, that it takes light from Earth to reach the sail at x is

$$\Delta t = t - t_e$$
$$= x/c$$

or $= x$ for c = 1 lt-yr/yr,

where t_e is the Earth time at which the light was emitted.

"Solving for t_e gives

$$t_e = t - x$$
$$= [1 + \sinh(a\,t') - \cosh(a\,t')]/a \quad .$$

"Replacing sinh and cosh with their exponential forms gives

$$t_e = [1 - \exp(-a\,t')]/a \quad .$$

"And solving for t' gives

$$t' = -[\ln(1 - a\,t_e)]/a$$ Q.E.D."

>Note that as the quantity $(a\,t_e)$ approaches 1, the quantity $-\ln(1 - a\,t_e)$ approaches >infinity; in other words, as the Earth time t_e approaches 1/a, the ship time t' approaches >infinity. If constant acceleration of the ship is maintained, communication from Earth to >the ship after Earth time 1/a is impossible. (For solution in conventional units, replace a >with a_{conv}/c, where a_{conv} is acceleration in conventional units.)

>My description [in Steve's 8/26/96 note] of the spacetime diagram that inspired this, for >those who are interested, was:
(See also note by Steve on 8/15/95.)

>The parametric equation [t, x] = (1/a) [sinh(a t'), (cosh(a t') − 1)] describes a >hyperbola with asymptotes t = x + 1/a and t = x − 1/a that are approached as t' goes to >infinity. So draw this hyperbola and its asymptotes on paper, putting t on the y-axis and >x an the x-axis; the hyperbola represents the worldline of the receiver. Also draw a >heavy line up the y-axis representing the worldline of the emitter. Now you can draw >lines with slope 1 (or parallel to the upper asymptote) between the worldline of the >emitter and the worldline of the receiver representing light rays sent from the emitter in >the direction of the receiver. Note that, because of this asymptote, rays that leave the >emitter after time 1/a can never reach the receiver (they all travel above the asymptote), >and that as the emitter time approaches 1/a, photons emitted quite close together in >emitter proper time are received with a much larger difference in receiver proper time.

B. ACCELERATION OF SAIL DRIVEN BY CONSTANT-POWER BEAM

(posted 9/11/96)
On 9/1 Steve wrote to me (and Timothy)–
>I wonder what the worldline would look like for an object that is accelerated by a >constant-output emitter? In other words, the emitter would send constant output power, >meaning the receiver would experience gradually decreasing received power and >acceleration as its proper time increases.

On 9/4 Kevin wrote to the Group–
>How about a mission which has a constant beam power, the acceleration would drop off >toward the turnaround point. In this case, the crew would start off with earth-like >gravity, and towards the middle of the trip, the gravity would be more lunar-like. ... The >advantage would be simplified beaming requirements, and the disadvantage would be a >slightly longer flight time.
>
>Questions:
>What would the top speed relative to Earth be?
>What is the total trip time. (crew time?)
>How much of this time is spent at less than $\frac{1}{2}$ g?

DERIVATION

For a power P_e sent out by an emitter, the power received by a sail (that captures the whole beam) which is receding at an apparent-velocity-ratio β is–

$$P_r = P_e \sqrt{1-\beta}\,/\sqrt{1+\beta}$$

....Doppler shift

$$= P_e\, \gamma\, (1 - \beta)$$

$$....\gamma = 1/\sqrt{1-\beta^2}$$

$= P_e\,[\cosh\theta - \sinh\theta]$

....$\gamma = \cosh\theta$; $\beta = \tanh\theta$ (definition of the velocity parameter θ)

$= P_e\, e^{-\theta}$

....using exponential forms of hyperbolic functions.

_____

Aside (to Timothy and Steve, 10/1/96):
I believe this Doppler relation, with $\theta = a\,t'$ for *constant acceleration* a, can be used to derive the logarithmic time dependence

$$t' = -[\ln(1 - a\,t)]/a$$

introduced by Steve in his note of 8/20 to the Group.

[On 8/26 Steve wrote (to me and Timothy)–
>...one can look at the relationship of received power to emitted power just as well in >terms of Doppler shifting as by using the relationship between emitter time and receiver >time...]

The desired derivation is as follows:

The received power P_r is constant for constant a. If the sail/ship is receding with a velocity parameter θ, which is $(a\,t')$, where t' is the elapsed ship time from start, then the emitted power P_e must be greater than the received power by the Doppler factor–

$$P_e = P_r\, e^{\theta} \quad .$$

If there is no spillage, the total energy received E_r must equal the total energy emitted E_e, or–

$$E_r = E_e = E \quad .$$

The differential reception of energy dE by the sail in differential sail time dt' is P_r, or–

$$dE/dt' = P_r \quad ,$$

and the differential emission of energy dE from the source in differential Earth time dt is P_e, or–

$$dE/dt = P_e = P_r\, e^{\theta} \quad .$$

Dividing the second differential relation by the first gives–

$$dt'/dt = e^{\theta}$$
$$= e^{a\,t'} \quad .$$

Grouping terms, we get–

$$dt = e^{-a\,t'}\,dt' \quad .$$

Integrating from t = 0 at t' = 0 to t and t', we get–

$$t = -(1/a)\,[e^{-a\,t'} - 1] \quad ,$$

or

$$a\,t = 1 - e^{-a\,t'} \quad .$$

Solving for t' gives–

$$t' = -\,[\ln(1 - a\,t)]/a \quad .$$ Q.E.D.

So, Steve's result for constant acceleration (Section V-A) can be obtained from consideration of the Doppler shift alone as he said. (Incidentally, I don't recall ever seeing the exponential relation for the Doppler shift before.)

_____

Back to the main derivation for constant emitted power:
(continuing 9/11 post)
So, for a sail receding at a velocity β which captures all the emitted beam, the received beam power is reduced by the Doppler shift according to–

$$P_r = P_e\, e^{-\theta} \quad .$$

(Note: If conversion of received power to retrothrust for the deceleration phase is required, then the sail must be a parabolic *reflector* to concentrate the beam power on a collector at the focal point. During the acceleration phase, however, the parabolic shape could be relaxed to flat, and the thrust from straight-back reflection would be the result of a momentum change of *twice* the incident momentum of the photons, i.e., $T = 2\,P_r/c$. In spite of the potential availability of this factor of 2, however, a conservative approach for the following analysis of the acceleration phase has been adopted, i.e., that thrust is only P_r/c. In the deceleration phase, the reflected photons will be absorbed by the collector, cancelling the momentum of reflection, and *forward* thrust will be $T = P_r/c$.)

The velocity-parameter equation of motion for a thrust, $T = P_r/c$, applied to a mass M is

$$T = M\,d\theta/dt' = P_r/c = P_e\, e^{-\theta}$$

....c = 1 lt-yr/yr,

which gives the "differential equation"

$$e^{\theta}\,d\theta = (P_e/M)\,dt' \quad .$$

Integrating from $\theta = 0$ at $t' = 0$, with P_e (and M) constant, gives–

$$(e^{\theta} - 1) = P_e\,t'/M \quad ,$$

so the description of the motion of the sail, in terms of the dependence of the velocity parameter θ on ship time t', is–

$$\theta = \ln[(P_e\,t'/M) + 1] \quad .$$

At the beginning of the flight, when thrust equals T_o, acceleration equals a_o and the received power P_r equals the emitted power P_e,

$$T_o = M\,a_o = P_e/c \quad ,$$

which leads to–

$$a_o = P_e/(c\,M) \longrightarrow P_e/M \quad \text{(for c = 1 lt-yr/yr)},$$

which makes the description of the motion of the sail–

$$\theta = \ln(a_o t' + 1) \quad .$$

For one space dimension in the direction of the destination,

$$dx/dt' = u = \sinh \theta$$
$$= \tfrac{1}{2} (e^{\theta} - e^{-\theta})$$
$$= \tfrac{1}{2} [(a_o t' + 1) - 1/(a_o t' + 1)] \quad .$$

Integrating this from x = 0 at t' = 0 gives–

$$x = \tfrac{1}{2} (\tfrac{1}{2} a_o t'^2 + t' - \theta/a_o) \quad .$$

The apparent (Earth) time t for the ship time t' is obtained from–

$$dt/dt' = \gamma = \cosh \theta$$
$$= \tfrac{1}{2} (e^{\theta} + e^{-\theta})$$
$$= \tfrac{1}{2} [(a_o t' + 1) + 1/(a_o t' + 1)] \quad .$$

Integrating this from t = 0 at t' = 0 gives–

$$t = \tfrac{1}{2} (\tfrac{1}{2} a_o t'^2 + t' + \theta/a_o) \quad .$$

The Earth time of emission t_e of the energy that arrives at the sail at t' is simply (for c = 1)–

$$t_e = t - x \quad .$$

The proper velocity u is given by–

$$u = \sinh \theta \quad .$$

The instantaneous proper acceleration, a, is given by the velocity-parameter equation of motion–

$$a = c \, d\theta/dt'$$
$$= a_o/(a_o t' + 1) \qquad \text{(for c = 1).}$$

RESULTS

Putting these relations together in the Fortran program COPOBM.FOR (included as an Addendum) gives the following values of θ, acceleration distance, proper velocity, instantaneous acceleration, Earth time for t' and Earth time of emission for reception at t', as a function of ship time t' for an initial acceleration of 1 g,

t'_{ship} (yr)	θ (rad)	Dist (lt-yr)	Prop Vel (lt-yr/yr)	Accel (g)	t_{Earth} (yr)	t_{emit} (yr)
0.0	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.5	0.4162	0.1130	0.4283	0.6595	0.5161	0.4031
1.0	0.7092	0.4146	0.7702	0.4920	1.1016	0.6870
1.5	0.9355	0.8776	1.0781	0.3924	1.7838	0.9062
2.0	1.1200	1.4900	1.3693	0.3263	2.5748	1.0848
2.5	1.2756	2.2453	1.6509	0.2793	3.4809	1.2356
3.0	1.4103	3.1399	1.9266	0.2441	4.5059	1.3660
3.5	1.5290	4.1712	2.1983	0.2168	5.6522	1.4810
4.0	1.6350	5.3377	2.4673	0.1949	6.9215	1.5837
4.5	1.7309	6.6382	2.7343	0.1771	8.3148	1.6766
5.0	1.8184	8.0718	2.9999	0.1623	9.8332	1.7613

For an example trip to a star 4.4906 (= 2 x 2.2453, if the deceleration distance is the same as the acceleration distance) lt-yr from Earth, the ship would accelerate for 2.5 ship years, reach a maximum proper velocity[1] of 1.6509 lt-yr/yr at an apparent (Earth) time of 3.4809 yr, and at turnover receive power emitted from Earth at 1.2356 yr after the departure date, giving an instantaneous acceleration of 0.2793 g.

These results confirm Kevin's intuitive estimates regarding acceleration levels. They also substantiate Steve's conclusion that the time of emission is limited (even for a constant-output emitter); from the table above, the duration of emission of the radiation accelerating a sail half way to a destination more than 16 lt-yr away (the last entry) is only about a year and three quarters.

The question remains to be answered: Is it possible for the ship to be decelerated in the second half of the distance?

(Note: This exercise may turn out to be academic if the inverse-square effects, causing expansion of the beam to be larger than the sail, are greater than the Doppler-shift effects.)

[1] For a proper velocity u the apparent velocity v is given by $v = u/\sqrt{1+u^2}$.

C. DECELERATION OF SAIL PUSHED BY CONSTANT-POWER BEAM

(posted 9/20/96)
The deceleration phase is not a mirror image of the acceleration phase. In the acceleration phase, the sail/ship is driven by the emitter beam's radiation pressure alone; in the deceleration phase, the radiation pressure of the emitter beam continues to provide a forward thrust, but the power of the emitter beam is collected and converted into a retro-thrust exhaust beam of, say, protons ejected forward to slow down the sail/ship.

This analysis considers a beam from a constant-output emitter with the same power that accelerated the sail/ship to a peak velocity half way to the destination star (per Section V-B, above). This analysis selects the exhaust velocity that brings the sail/ship to a halt in the remaining half of the distance. (An earlier analysis by Timothy on 3/29/96 considered the problem of decelerating the sail/ship at a constant rate, not with a constant emitter-beam power, with an exhaust velocity selected to minimize the energy consumption.)

ANALYSIS (adapted and expanded from Timothy's 3/29 analysis)

Following the notation of the previous section, the power radiated from the emitter is P_e, and the power received by the sail capturing all the emitter beam is P_r. The thrust exerted by the radiation pressure is P_r/c. At the start of the acceleration phase when $P_r = P_e$, the mass of the sail/ship is M_o, the initial acceleration is a_o, and the required emitted power is–

$$P_e = T\,c$$
$$= M_o\,a_o\,c \quad . \qquad [\text{for } a_o = 1\text{ g},\ a_o\,c = 2{,}940\text{ Mw/kgf}]$$

When the sail/ship has been accelerated to an apparent-velocity-ratio β, having the velocity parameter θ from the defining relation $\beta = \tanh\theta$, the received power is reduced by the Doppler shift according to–

$$P_r = P_e\,e^{-\theta} \quad ,$$

and the thrust T_b of the received beam's radiation pressure is–

$$T_b = P_r/c$$
$$= (P_e/c)\,e^{-\theta}$$
$$= M_o\,a_o\,e^{-\theta} \quad .$$

The emitter-beam power is collected and converted with an efficiency η to the power P_{ex} of a retro-thrust exhaust beam, or–

$$P_{ex} = \eta\,P_r \quad .$$

The exhaust-beam power is the rate of ejection of kinetic energy, or–

$$P_{ex} = (dM/dt')\,(\gamma_{exh} - 1)\,c^2 \quad ,$$

where (dM/dt') is the rate of ejection of propellant mass (made up of protons, say), and γ_{exh} is the exhaust beam's energy factor, $\gamma_{exh} = 1/\sqrt{1 - V_{exh}^2}$, derived from the beam's exhaust velocity V_{exh} (lt-yr/yr).

The thrust, T_{ex}, of the exhaust beam is given by–

$T_{ex} = (dM/dt')\ \gamma_{exh}\ V_{exh}\ c$.

Substituting the dM/dt' derived from the equation for P_{ex} above gives–

$T_{ex} = [P_{ex}\ \gamma_{exh}\ V_{exh}\ c]/[(\gamma_{exh} - 1)\ c^2]$

$= \eta\ (P_r/c)\ RBE$,

(posted 3/16/01 in review)

where the Relative Beam Effectiveness

$RBE = \gamma_{exh}\ V_{exh}/(\gamma_{exh} - 1)$ $\left[= U_{exh}\Big/\left(\sqrt{1 + U_{exh}^2} - 1\right)\ ;\ = \sqrt{1 + \left(2mc^2 / MeV\right)}\ \text{(p. 31)}\right]$.

The value of RBE is 1.0000 for photons (either collected emitter-beam photons or exhaust-beam photons) and highly relativistic electrons, but for protons, which have lower velocity, RBE is greater than 1 and grows as the exhaust velocity V_{exh} (in lt-yr/yr) is decreased, according to the following table:

V_{exh}	RBE
1.0	1.0000
0.9	1.5954
0.8	2.0000
0.7	2.4488
0.6	3.0000
0.5	3.7321
0.4	4.7913
0.3	6.5131
0.2	9.8990
0.1	19.9499

(back to 9/20/96 post)

Note that the exhaust-beam retro-thrust exceeds the radiation-pressure thrust, making deceleration of the sail/ship possible, when η x RBE is greater than 1. For $V_{exh} = 0.9$, for example, RBE is 1.5954, and deceleration is possible only if the efficiency η of conversion of received power to exhaust power is greater than 1/1.5954 = 0.6268.

[The efficiency required for deceleration of the emitter-beam-pushed sail is easier and easier to achieve as the exhaust velocity of the protons is made lower and lower, to be balanced, however, against a higher and higher required mass ratio.]

The acceleration, a, of the sail/ship (negative, hopefully) is given by–

$a = (T_b - T_{ex})/M$

where M is the mass of the sail/ship at the ship time t'.

The rate of change of the mass of the sail/ship as propellant is ejected is obtained from the T_{ex} equation above–

$$dM/dt' = T_{ex}/(\gamma_{exh}\, V_{exh}\, c) \quad .$$

The rate of change of the velocity parameter is given by the velocity-parameter equation of motion–

$$d\theta/dt' = a/c$$
$$= (T_b - T_{ex})/(M\, c) \quad .$$

We thus have two simultaneous differential equations, with dM/dt' involving $e^{-\theta}$ through the dependence of T_{ex} on P_r, and with $d\theta/dt'$ involving 1/M. The coupling therefore is non-linear, and the method of solution I find in my math book is for linear simultaneous differential equations. Not being a "mathochist" (one who enjoys suffering in the solution of higher math problems), I chose to integrate these equations numerically; see the Fortran program SAILTRIP, included as an Addendum. The implemented difference equations are–

$$M(n+1) = M(n) - [T_{ex}(n)/(\gamma_{exh}\, V_{exh})]\, [t'(n+1) - t'(n)]$$
$$\theta(n+1) = \theta(n) + [(T_b(n) - T_{ex}(n))/M(n)]\, [t'(n+1) - t'(n)] \quad .$$

(The SAILTRIP program also includes the COPOBM acceleration-phase calculation outlined in Section V-B, allowing the program to cover the whole trip from start to destination.)

Results become consistent to better than three significant figures for time steps, [t'(n+1) – t'(n)], smaller than 0.01 yr, with insignificant computation time (about one second for the deceleration phase) for a time step of 0.001 yr. The deceleration phase is repeated with trial values of the exhaust velocity until interpolation yields the desired deceleration distance within a tolerance of 0.0001 lt-yr.

RESULTS

Calculations with the SAILTRIP program were made for a trip to τ Ceti, whose distance was taken to be 11.9 lt-yr. It was found that, with the constant-output emitter, the deceleration grows to exceed 1 g near the destination, where the thrust increases as the velocity and therefore the Doppler shift decrease, and where the mass of the sail/ship decreases as the propellant is depleted. The deceleration is limited in the calculation to 1 g by the simple expedient of furling the sail (first suggested by Kelly on 4/3/96).

The calculated values of θ, distance, proper velocity, acceleration, apparent (Earth) time, time of emission of radiation from Earth and mass ratio (ratio of initial mass to the instantaneous mass), as functions of ship time t', are given in the following table for the τ Ceti trip, for a conversion efficiency of received power to exhaust power of 1.0. Also stated are the values of the exhaust velocity, in lt-yr/yr, that gives the desired deceleration

distance, with the corresponding kinetic energy, in MeV, of protons having that velocity, and of the final relative area of the furled sail.

t'_{ship} (yr)	θ (rad)	Dist (lt-yr)	Prop Vel (lt-yr/yr)	Accel (g)	t_{Earth} (yr)	t_{emit} (yr)	Mratio
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	1.0000
0.5000	0.4162	0.1130	0.4283	0.6595	0.5161	0.4031	1.0000
1.0000	0.7092	0.4146	0.7702	0.4920	1.1016	0.6870	1.0000
1.5000	0.9355	0.8776	1.0781	0.3924	1.7838	0.9062	1.0000
2.0000	1.1200	1.4900	1.3693	0.3263	2.5748	1.0848	1.0000
2.5000	1.2756	2.2453	1.6509	0.2793	3.4809	1.2356	1.0000
3.0000	1.4103	3.1399	1.9266	0.2441	4.5059	1.3660	1.0000
3.5000	1.5290	4.1712	2.1983	0.2168	5.6522	1.4810	1.0000
4.0000	1.6350	5.3377	2.4673	0.1949	6.9215	1.5837	1.0000
4.2418	1.6825	5.9500	2.5967	0.1859	7.5797	1.6297	1.0000
Exhaust Velocity = 0.88301; Proton MeV = 1060.5							
4.5000	1.6483	6.6085	2.5029	–0.1337	8.2871	1.6786	1.0467
5.0000	1.5731	7.8121	2.3070	–0.1593	9.5904	1.7783	1.1570
5.5000	1.4818	8.9119	2.0867	–0.1971	10.7986	1.8867	1.3067
6.0000	1.3656	9.8934	1.8314	–0.2584	11.9003	2.0069	1.5254
6.5000	1.2058	10.7345	1.5199	–0.3750	12.8791	2.1446	1.8873
7.0000	0.9482	11.3959	1.0968	–0.6833	13.7090	2.3132	2.6602
7.5000	0.4669	11.7924	0.4840	–1.0011	14.3512	2.5588	5.0527
7.9516	0.0000	11.9000	0.0000	–1.0003	14.8195	2.9195	9.4145
Final sail furl = 0.160							

Furling the sail to limit the deceleration to 1 g begins about 0.7 yr before arrival at the destination. Even with furling, the deceleration time is shorter than the acceleration time for the same distance. The average deceleration is greater than the average acceleration because the decreasing mass in the deceleration phase overrides the reduction in thrust that comes from the competition between radiation-pressure push and exhaust retrothrust.

Even though the trip takes about 8 years of ship time, or about 15 years of Earth time, the total job of the emitter is over in less than 3 Earth years (consistent with Steve's original prediction, even including the deceleration phase).

There may be performance gains from changing the turnover point from the halfway point. For example, as the turnover point is moved earlier than the halfway point, a higher exhaust velocity (lower RBE and lower thrust) is allowed, which, together with the lower peak velocity, calls for a lower mass ratio.

The performance results tabulated above are for 100 percent conversion efficiency ($\eta = 1.0$) from received power to exhaust power. The effects of reduced conversion efficiency ($\eta < 1$) on required exhaust velocity, final sail furl and, most importantly, required mass ratio are given in the following table:

η	exhaust velocity	final sail furl	mass ratio
1.0	0.883	0.160	9.41
0.9	0.849	0.105	15.44
0.8	0.809	0.060	29.05
0.7	0.760	0.029	67.57

Producing high efficiency of conversion from received power to exhaust power may be as challenging (and as crucial to the success of the concept) as constructing the emitter or the sail.

D. CURRENT FOR ELECTRIC RETRO-THRUSTER FOR SAIL

(posted 9/26/96)
On 9/20 Kelly wrote (regarding "Deceleration of sail pushed by constant-power beam")–
>If we assume electric acceleration of the particles that's also very efficient, then how >large a drive system would we need to get that degree of acceleration?

The efficiency of accelerators that I was familiar with as a graduate student was not very high; a 1 microamp proton current at 30 million volts (30 watts output) was the best I can remember, while the input power was about 300 kilowatts. This gives an efficiency of only 0.01 percent. (Even if my memory is off by 2 orders of magnitude the right way, the efficiency would still be pretty poor by our standards.) For nuclear-research accelerators, there never has been a strong need for high current; a microamp gives lots of particles. Some of you may be more familiar than I am with contemporary accelerators.

We can calculate the proton current required to give an appreciable thrust from relations in Section V-C. Using that Section's notation, the relation for the received power is–

$$P_r = M_o\, a_o\, c\, e^{-\theta} \quad ,$$

and the exhaust power, $P_{ex} = i\, V$ (current times voltage), is given by–
(note: for $a_o = 1$ g, $a_o\, c = 2{,}940$ Mw per kg thrust)

$$P_{ex} = \eta\, P_r$$
$$= \eta\, M_o\, a_o\, c\, e^{-\theta}$$
$$= \eta\, M_o\, (2{,}940 \text{ Mw/kgf})\, e^{-\theta} \quad .$$

Now the exhaust power will be increasing as θ is decreasing, in the deceleration phase, until the maximum allowable deceleration of 1 g is reached. Thereafter the power will be decreased by furling the sail as the mass decreases, to keep the deceleration at 1 g. The maximum exhaust power will therefore occur just as the deceleration reaches 1 g while the sail is still fully open. For the τ Ceti mission history tabulated in Section V-C, the value of θ when 1 g is first reached is about 0.78. For $\eta = 1.0$, the maximum proton current i, for the calculated required proton energy of 1,060 million volts, is–

$$i = [M_o\, (2{,}940 \text{ Mw/kg})\, e^{-0.78}]/1{,}060 \text{ megavolts}$$
$$= 1.27 \text{ amps per kg of initial sail/ship mass x } M_o \quad .$$

So, we're talking about roughly 1 amp of 1 GeV protons for each kg of initial sail/ship mass. That current is to be compared with the approximately 1 microamp of proton-accelerator current of today's state of the art. (Check that value; I'd be surprised if you find much higher currents than 1 microamp at 1 GeV today.)

So, we need remarkable progress in improving both proton current and conversion efficiency before we can count on the beam-driven sail (or any other propulsion system requiring an electric thruster).

E. REQUIRED MICROWAVE-ANTENNA SIZE

(posted 10/24/96)
On 10/21/96 Kevin wrote–
>I really think this is the best design we've come up with yet. Aside from the cost and >political will issues, none of this technology is beyond our capability. We know how to >make solar collectors, masers, linear accelerators and closed system ecologies.
>
>The question remains, can we build [the solar collector] large enough, [the maser array] >precise enough, [the linac] efficient enough and will [the closed system] last long enough >to make it to τ Ceti and back again. But then, these are engineering problems, not >physics problems.

I think Kevin's note from which the above quote was taken is a very good wrap-up of the beam-driven-sail concept, except that the calculated results that he draws from Section V-C were qualified by the condition that the sail captures all the emitter beam. At the end of Section V-B, I wrote–
"(Note: This exercise may turn out to be academic if the inverse-square effects, causing expansion of the beam to be larger than the sail, are greater than the Doppler-shift effects.)"

Kevin acknowledged this issue when he wrote, on 10/21–
>This is one of the problems that's worried Tim. Tim fears that the beam will spread out >too much. I think that the beam can be focused into a nearly parallel stream of photons; >thus there will be no spreading and diffusion. *IF* this can be achieved, then range won't >be a problem at all. The ship will be able to travel to any star within a good 100 light >years on a 3-4 year pulse of maser energy. Since most of a 100 light-year trip will be >spent at insanely high fractions of c, I expect the one-way trip time to be on the order >of ten years for the crew. Of course a return mission would arrive a good 250 years >after departure.

The above quotes indicate that there are some uncertainties that need to be cleared away with calculations of (1) numbers regarding antenna-size requirements to avoid too much beam attenuation due to inverse-square effects, and (2) numbers regarding a 100-lt-yr trip.

1. ANTENNA SIZE REQUIREMENT (A tutorial; read at your own risk.)

In the "far field" [range > $D_e^2/(2.44\ \lambda)$, to be shown below] of the emitting aperture, focusing to produce a convergent beam (or even "a nearly parallel stream of photons") is not possible, and Fraunhofer diffraction (constant angular beam width, producing linear divergence of beam width or inverse-square dependence of power per unit area, with distance) applies. A diffraction-limited beam ("perfect" antenna shape) contains about 84 percent of its power within the main lobe which has an angular radius of $1.22\ \lambda/D_e$ (where

λ is the wavelength of the emitted radiation and D_e is the diameter of the emitting aperture).

In the "near field," Fraunhofer-diffraction considerations are not applicable for defining the scaling of beam-cross-section ("spot") size with distance. But focusing to produce a convergent beam is possible, and we can use simple geometrical-optics considerations to determine the focused-spot size. For a reflecting beam-forming "mirror" (emitting antenna) with a focal length f the distance from the mirror to the image spot, d_i, is related to the distance from the mirror to the object, d_o, by the relation–

$(1/d_i) + (1/d_o) = (1/f)$,

and the magnification, i.e., the ratio of the image size to the object size, is given simply by the ratio d_i/d_o. In our case, the distance from the mirror to the image spot (on the sail) is many, many focal lengths of the mirror, so d_o does not need to be changed much from the focal length f to compensate for the change in d_i as the sail moves away from the emitter. Therefore the focused image spot size (obtained from the magnification d_i/d_o, with essentially constant d_o) in effect increases directly with d_i (for constant object size), resulting in an inverse-square dependence of power per unit area again.

[But the image spot size is smaller than the mirror up to the boundary between the near and far fields, so, when the sail is larger than the image spot size, there will be no distance dependence of the power captured by the sail.]

The range boundary between the "near" and "far" fields can be defined as the distance at which the focused image spot size is equal to the size of the beam-forming aperture, and at that range the spot size is that given by Fraunhofer diffraction (with a factor of 2 to go from radius to diameter), so–

spot diameter = D_e = range x 2 x 1.22 x λ/D_e ,

or, solving for the range that gives the boundary between the near and far fields,

range = $D_e^2/(2.44\ \lambda)$. Q.E.D.

If the image spot size at the boundary is that given by Fraunhofer diffraction, and the spot size has the same dependence on distance in the near field as the Fraunhofer spot has in the far field, then we can use the Fraunhofer angular width to determine the spot size in the near field. (What we can't do in the near field is take the same shape of the power distribution within the spot. The power distribution within the spot is governed by "Fresnel" diffraction, which we'll assume gives a gross distribution not enough different to bother us at this point.)

If we set the Fraunhofer-diffraction angular width equal to the angle subtended by the sail diameter D_s at the range R, we can get an expression for the diameter of the emitting antenna D_e that is required to put "84 percent" of the emitted power within the area of the sail:

$D_e = 2.44\ \lambda\ R/D_s$.

For microwaves with λ = 1 cm and a sail diameter of 1000 km, say, the required diameter of the emitting antenna grows 231,000 km for each light-year of distance to the sail, to keep the main lobe just within the sail diameter. This is for an antenna whose figure (shape) is correct to a fraction of a wavelength.

For an emitting-antenna diameter of 231,000 km (and wavelength of 1 cm) for a 1-lt-yr distance to the sail, the boundary between near and far fields is at a range of about 231 lt-yr, far enough away to say we're operating in the near field.
[Check: at 1 lt-yr distance, the spot size is (1 lt-yr/231 lt-yr) x 231,000 km = 1000 km, the diameter of the sail, as desired.]

Are we prepared to build an expanding antenna this large? It looks as if the driving beam should be a laser instead of a maser. For any wavelength, however, figure control is going to remain a challenge.

2. DATA FOR 100-LIGHT-YEAR TRIP
(in spite of the foregoing; i.e., spillage of the beam is somehow avoided)

A run with the program SAILTRIP (see Addendum) with DSTAR = 100 lt-yr gives the following data for the one-way trip:

Proper (ship) time = 23.8473 yr
Apparent (Earth) time = 104.2934 yr
Total Earth time for emission = 4.2934 yr
Peak proper velocity = 7.2603 lt-yr/yr
Peak apparent velocity = 0.9906 lt-yr/yr
Minimum acceleration (at turnover) = 0.0685 g
Exhaust velocity for deceleration = 0.8935 lt-yr/yr
Mass ratio for deceleration (η = 1.0) = 33.62

The trip time for the crew is not as short as Kevin estimated ("ten years") because the acceleration is so low for most of the trip. His estimates for emission time ("3-4 ... year"), Earth time for round trip ("a good 250 years") and speed ("insanely high fractions of c") are pretty good.

F. BEAM PROPERTIES, CONTROL, ETC.

(posted 4/7/96)
A laser beam with wavelength λ that has been formed by a primary objective mirror with an aperture diameter D will diverge in the far field [where you'll be if the range is greater than $D^2/(2.44\ \lambda)$] due to diffraction. The angular width of a "diffraction-limited" beam at half maximum is the same as the half-width from the peak to the first null, which is the Rayleigh criterion, $1.22\ \lambda/D$, for angular resolution of two closely spaced sources.

This width is further broadened due to (1) thermal effects in the lasing medium, by a factor called "beam quality" that is in the range 1.1-1.3, depending on the type of laser, and (2) beam-pointing instability called "jitter," usually about one-third the diffraction-limited beam divergence, which enters in a root-sum-squares way. The overall angular beam width of a contemporary laser "weapon" is of the order of a microradian.

(posted 4/11/96 to Lee)
A laser beam conforms with the inverse-square law because its cross-sectional area, for a fixed angular width, increases as the square of the distance. The attenuation in interstellar space due to absorption by matter in the beam and scattering out of the beam by that same matter is generally a small effect compared with the inverse-square-law dissipation of the beam power per unit target area.

A laser beam of yellow light (λ about 0.6×10^{-6} m) from an objective mirror with a diameter D of 400 km (your number) would have a diffraction-limited angular divergence ($1.22\ \lambda/D$) of about 1.8×10^{-12} radian. (We ignore here problems with maintaining the figure of a 400-km-diameter mirror to within a fraction of the wavelength of the laser light.) At a distance of ten light-years (9.46×10^{13} km), say, the linear width of this laser beam would be about 170 km. For a mirror diameter of 40 km, the beam width at ten light-years would be 1700 km; for 4 km, 17,000 km. In the middle case, a sail with a diameter of 100 km, say, would capture $(100/1700)^2$, or about 1/300th, of the beam power (assuming, for simplicity, a constant power per unit area across the beam), or would capture about 1/3rd of the beam power at one light-year distance. The plausibility of a laser-beam-driven-sail propulsion system therefore seems to hinge on the feasibility of making a many-km-diameter laser objective mirror with a surface precision of a fraction of the wavelength of light and/or making a laser with a power orders of magnitude greater than the sail would use.

(posted 10/30/96)
On 10/24 Kelly wrote–
>Questions:
>How much range deviation can you tolerate? Since the beam is being aimed at a ship >you can't see in real time, you'd have to expect it would drift ahead or behind the exact >focus spot. How much slack is allowed?
>

>What is the lateral deviation of the beam? I.e., what's the power per m^2 in the center vs >the edge of the focused spot? The differences would distort the sail and alter the ship's >course and acceleration.
>
>As mentioned above, how precisely can we measure the position of an object floating in >space? Assuming each microwave emitter platform has laser ranging info to each/some >of the others. Can you get the necessary positional accuracy? (Within a couple mm?) >Note you don't need to control the position that accurately, just know what it is so you >can compensate for it.

On 10/24 Zenon wrote–
>However, if the platform moves along the Sun-centered orbit, its velocity is of the order >of tens of kilometers a second..., hence it must compensate for its change of position >with appropriate change of orientation, and the latter must be VERY accurate...

On 10/26 Timothy wrote–
>I'd rather turn things around, not let the emitter follow the ship, but let the ship follow >the emitted beam [first suggested to this Group by Kevin on 2/6/95 at 12:04 CST]. The >beaming station makes a "focus" (as far as interference allows) and beams it straight >forward (in the direction of τ Ceti). In this case it's not the velocity of the orbiting >station that is important, but its acceleration (to the center of gravity), which is rather >low, low enough for the starship to compensate and change its direction.

On 10/28 Zenon wrote–
>That is, the ship must go along the helical curve with the radius equal to the radius of the >beaming station's orbit (assuming the plane of the orbit is perpendicular to the direction >of τ Ceti), or along a sinusoid with amplitude equal to the diameter of the orbit >(assuming the direction to τ Ceti lies within the plane of the orbit).
>...
>However, I wonder if the jiggle of the direction of the beam due to "directional noise" >can be compensated in this way...

On 10/28 Timothy wrote–
>What if
> $\lambda = 500$ nm $= 5 \times 10^{-7}$ m, (Bluish green)
> R = 10 lt-yr $= 9.46 \times 10^{16}$ m
> $D_s = 300$ km $= 3 \times 10^5$ m
>Then [to stay in the near field at 10 lt-yr]
>D_e = ... = 385 km.
>300 and 385 kilometer diameters don't seem like a headache...

In recapitulation, questions/observations above address issues of–
1. Depth of focus
2. Power distribution in beam spot
3. Antenna figure sensing/control

4. Pointing control
   a. Direction: to ship or to destination
   b. Orbital motion of antenna platform
   c. "Jitter"
5. Sail/ship guidance
6. Effects of reduced wavelength

To initiate thinking on these issues, some tentative introductory answers and opinions in the order itemized above are–

1. Depth of focus

Assume the beam from a 231,000-km-diameter microwave antenna is focused to a sail (beam-spot) diameter of 1000 km at a distance of 1 lt-yr. At the distance from the focus that the beam cross-section has grown from the area at the focus by 10 percent (power per unit area reduced by about 10 percent), say, the diameter of the beam has grown by 1000 km x $[\sqrt{1+0.1} - 1]$, or about 49 km. The distance from the focus for that growth is (49/231,000) x 1 lt-yr, or approximately 2 x 10^{-4} lt-yr.

So, the sail/ship has to stay within about 2 x 10^{-4} lt-yr of the focus to keep the power from dropping off by more than 10 percent.

2. Power distribution in beam spot

If the sail/ship is in the near field of the beam-forming aperture, then the radial power distribution is given by Fresnel diffraction: the envelope of the intensity across the spot is essentially "flat" (the profile is rectangular), enclosing alternating narrow concentric rings of dark and light. As the sail approaches the boundary to the far field, the radial distribution of intensity morphs into the approximately-Gaussian profile of the Fraunhofer diffraction pattern. I wouldn't want to undertake the calculation of the actual power distribution at any arbitrary range. Assuming it's flat for our purposes is probably adequate.

3. Antenna figure sensing/control

I don't have any detailed information at my fingertips in this area, but what superficial information I do have indicates that this problem, while challenging, should not be a show-stopper. (A lot of work has been done on phased arrays of both antennas and mirror segments; some of it pertains to fine beam steering, also.) I believe this problem is of far lesser import than achieving the required large sizes of antennas/mirrors.

4. Pointing control
   a. Direction: to ship or to destination

I believe Timothy has made the case for pointing the beam at the destination star quite well. Let the sail/ship steer to stay in the beam (see also below).

b. Orbital motion of antenna platform

As Timothy calculated in his 10/29 note for a beaming station in Solar orbit at the distance of the Earth, the maximum lateral acceleration to follow a helical or sinusoidal interstellar path of that amplitude and period would be about 6 x 10^{-4} gs. This means that the sail must control its attitude (tilt) through not much more than about 10^{-3} radians to follow the desired path. [The maximum pointing deflection for the beaming station is the parallax of the destination star, or no more than the 0.76″ for the nearest star, α Centauri.]
[(posted 5/7/96 to Timothy)
Pointing from a moving platform is "old hat." Readjusting a beam on a star from a platform in Keplerian motion should be not much different from taking a long-exposure photograph with the Hubble telescope. An early development goal for tracking accuracy (guidance stability) of the Hubble telescope was 0.005 arc second (0.025 microradian) for exposure periods up to 10 hours (the goal for angular resolution was 0.05 arc sec).]
[Note: the diameter of the disk of α Centauri A is about 0.03 microradian.]

c. "Jitter"

One of the losses to be considered in depositing a lethal fluence from a laser weapon on a target is the effect of fairly-high-frequency, small-amplitude oscillatory motions of the beam axis about the desired direction, called "jitter" (Zenon's "jiggles," or "directional noise"). The frequencies are generally higher than about 100Hz, and the amplitudes, with today's technology, are a fraction of a microradian [see the note above for the Hubble telescope's guidance stability]. This motion is the result of such things as pointing-control limit cycles and mechanical vibrations. This is a random process that lends itself to averaging. The intensity at any point on the spot is given by folding together the beam profile and a time-average of the deflection vs time of the beam direction. On the other hand, with extremely large antenna/mirror arrays, the frequency may be so low that deflections could be measured in real time. These measurements could be introduced as correction inputs to an electronic fine-pointing-control (element-phasing) system for the array.

5. Sail/ship guidance

The sail/ship can have outriggers beyond the edge of the sail to sense the edge of the beam and provide steering-correction inputs to adjust the tilt of the sail to stay at the radial center of the beam.

The acceleration of the sail/ship will depend on its distance from the focus. (Operating in the near field has some advantages). A computer simulation of the sail/ship's motion based on a power level reduced by a safety factor from that at the focus can be made to provide a projected safe position of the sail/ship at any time.

If the focus is placed behind the projected safe position of the sail/ship during the actual flight, any lag in acceleration of the sail below the "safe" level will drop it back closer to the focus where the power is greater. The higher power there will drive it forward again toward the safe position, in a stable control condition at a power level between full (at the focus) and "safe."

6. Effects of reduced wavelength

While reducing the wavelength of the radiation in the beam to that of blue-green light allows the required emitting aperture to be reduced to "only" 385 km [to keep a 300-km-diameter "sail" in the near field at 10 lt-yr distance], the "antenna" becomes a mirror. A mirror has the problem (among others) of maintaining its reflectivity over long periods in the presence of hazards of the space environment such as micrometeoroid erosion.

In partial answer to Timothy's question of 10/29 regarding efficiency of lasers, I seem to recall efficiencies of about 30 percent, 10 percent and 5 percent for chemical, solid-state and gas-dynamic lasers, respectively. (The only mention of the efficiencies of lasers in the Grolier Encyclopedia is a statement that CO_2 electric lasers have efficiencies in the 15-30 percent range; I don't have any references for any other lasers than chemical lasers, the favorite of weapons developers.)

Conversion of received light energy to electrical energy (to power the deceleration thruster) at any reasonable efficiency probably involves a thermodynamic cycle. (Heat engines are about twice as efficient as the best solar cells.)

ADDENDUM: USEFUL CONSTANTS

(from Handbook of Chemistry and Physics, 77th Edition, 1996-7)

velocity of light $c = 299{,}792{,}458$ m/sec = 1 lt-yr/yr

electronic charge $e = 1.602\ 177\ 33 \times 10^{-19}$ coulomb

1 MeV $= 1.602\ 177\ 33 \times 10^{-13}$ Joule

mass of proton $m_p = 1.672\ 623\ 1 \times 10^{-27}$ kg $= 1.007\ 276\ 470$ amu

$= 938.272\ 31$ MeV

1 amu $= 1.660\ 540\ 2 \times 10^{-27}$ kg $= 931.494\ 32$ MeV

proton/electron $m_p/m_e = 1{,}836.152\ 701$

mass:force conversion $g_c = 9.806\ 65$ m/sec^2

constant of gravitation $G = 6.672\ 59 \times 10^{-11}$ m^3/(kg sec)

astronomical unit $AU = 1.495\ 978\ 70 \times 10^{11}$ m

year yr = 1/100 of Julian century = 365.25 d $= 3.15576 \times 10^7$ sec

light-year lt-yr $= 9.460\ 730 \times 10^{15}$ m

1 lt-yr/yr$^2 = 9.499\ 850$ m/sec^2

parsec pc $= 3.085\ 677\ 6 \times 10^{16}$ m $= 3.261\ 563\ 8$ lt-yr

Avogadro's number $N_A = 6.022\ 136\ 7 \times 10^{23}$ /mol

Stefan-Boltzmann const $\sigma = 5.670\ 51 \times 10^{-8}$ W/(m^2 K^4)

solar constant at Earth $s = 1{,}373$ W/m^2

ADDENDUM: STARS WITHIN 16 LIGHT YEARS OF THE SUN
(submitted by Lee 6/21/97)

NAME	DISTANCE (lt-yr)	SPECTRAL TYPE	APPARENT MAGNITUDE	ABSOLUTE MAGNITUDE
Sun	N/A	G2 V	–26.72	4.85
Proxima Centauri	4.24	M6 V	11.05	15.49
α Centauri A	4.34	G2 V	–0.01	4.37
α Centauri B	4.34	K1 V	1.33	5.71
Barnard's Star	5.97	M4 V	9.54	13.22
Wolf 359	7.44	M6 V	13.53	16.65
BD + 36°2147	8.22	M2 V	7.5	10.5
L-726-8 A	8.41	M6 V	12.52	15.46
L-726-8 B	8.41	M6 V	13.02	15.96
Sirius A	8.64	A1 V	–1.46	1.42
Sirius B	8.64	White Dwarf	8.3	11.2
Ross 154	9.5	M4 V	10.45	13.14
Ross 248	10.37	M5 V	12.29	14.78
ε Eridani	10.8	K2 V	3.73	6.14
Ross 128	10.93	M4 V	11.1	13.47
61 Cygni A	11.1	K4 V	5.22	7.56
61 Cygni B	11.1	K5 V	6.03	8.37
ε Indi	11.22	K3 V	4.68	7
BD + 43°44 A	11.25	M1 V	8.08	10.39
BD + 43°44 B	11.25	M4 V	11.06	13.37
L786-6	11.25	Uncertain	12.18	14.49
Procyon A	11.45	F5 V-IV	0.37	2.64
Procyon B	11.45	White Dwarf	10.7	13
BD + 59°1915	11.58	M3 V	8.9	11.15
BD + 59°1915 B	11.58	M4 V	9.69	11.94
CD – 36°15693	11.68	M1 V	7.35	9.58
G51-15	11.7	M7 V	14.81	17.03
τ Ceti	11.77	G8 V	3.5	5.72
BD + 5°1668	12.26	M4 V	9.82	11.94
L725-32	12.49	M4 V	12.04	14.12
CD – 39°14192	12.56	M6 V	6.66	8.74
Kapteyn's Star	12.75	MO V	8.84	10.88
Kruger 60 A	12.88	M3 V	9.85	11.87
Kruger 60 B	12.88	Uncertain	11.3	13.3
BD – 12°4253	13.21	M4 V	10.11	12.07
Ross 614 A	13.24	M4 V	11.1	13.12
Ross 614 B	13.24	Uncertain	14	16
van Maanen's Star	14.06	White Dwarf	12.37	14.2
Wolf 424 A	14.19	M5 V	13.16	14.97
Wolf 424 B	14.19	Uncertain	13.4	15.2

CD – 37°15492	14.48	M2 V	8.56	10.32
L1159-16	14.55	M4 V	12.26	14.01
LP731-58	14.91	Uncertain	15.6	17.3
CD – 46°11540	15.10	M3 V	9.37	11.04
G158 - 27	15.23	M5	13.99	15.61
CD – 49°13515	15.23	M2 V	8.67	10.32
CD – 44°11909	15.30	M4 V	10.96	12.6
BD + 68°946	15.30	M3 V	9.15	10.79
G208 - 44 A	15.46	Uncertain	13.41	15.03
G208 - 44 B	15.46	M5	13.99	15.61
BD – 15°6290	15.59	M4 V	10.17	11.77
o2 Eridani A	15.75	K1 V	4.43	6.01
o2 Eridani B	15.75	White Dwarf	9.52	11.1
o2 Eridani C	15.75	M4 V	11.17	12.75
BD + 20°22465	15.82	M3 V	9.43	11
L145-141	15.82	White Dwarf	11.5	13.07
70 Ophiuchi A	16.08	KO V	4.22	5.76
70 Ophiuchi B	16.08	K4 V	6	7.54
BD + 43°4305	16.3	M5	10.2	11.7

ADDENDUM: Fortran Programs used to Perform the Calculations

The Fortran programs are included here to serve primarily as ASCII renderings of the equations, proof-read by a computer.

Program OPTUEXH

For an input value of conversion efficiency η, and for each value of U_{end}: 0.2, 0.5, 1.0, 2.0, 3.0, 4.0 and 5.0, the program calculates the V_{end}, the γ_{end} and the θ. For each U_{end} the program then calculates the eff for values of U_{exh} increasing from 0.01 by factors of 1.01, until the next value of eff is less than 1.00001 times the previous value. If the next value is less than the preceding value, a maximum is passed; if it is not less than the preceding value (i.e., if it is between 1.00000 and 1.00001 times the preceding), the calculation is stopped and the last value of U_{exh} is called the "optimum." When a maximum is passed, the optimum U_{exh} is calculated by fitting a second-degree curve through the three points that include the maximum. The value of the maximum is calculated from the optimum U_{exh}. The ratios of initial antimatter mass to M_{bo} and M_i are derived from expressions in Sections II and III, as–

$$M_a/M_{bo} = (r - 1)(\gamma_{exh} - 1)/[\gamma_{exh} - 1 + \eta\ (1 + DUMP)]$$
$$eff = (\gamma_{end} - 1)\ [\gamma_{exh} - 1 + \eta\ (1 + DUMP)]/[\eta\ (r - 1)(\gamma_{exh} - 1)]$$
$$= (\gamma_{end} - 1)/(\eta\ M_a/M_{bo})$$
$$M_a/M_{bo} = (\gamma_{end} - 1)/(eff\ \eta)$$
$$M_{am} = \tfrac{1}{2}\ M_a$$
$$M_{am}/M_{bo} = (\gamma_{end} - 1)/(2\ eff\ \eta)$$
$$M_{am}/M_i = (M_{am}/M_{bo})(M_{bo}/M_i)$$
$$= (M_{am}/M_{bo})/r$$
$$= (\gamma_{end} - 1)/(2\ eff\ \eta\ r)$$

```
      PROGRAM OPTUEXH                    !5/1/96, 8/8/96
C     Charge Neutralization 1/31/97; OPTU 2/1
  101 FORMAT(2X, 26H Conversion Efficiency = ?)
  102 FORMAT(2X, 4HUend, 7H  Vend , 8H optVexh, 11H    optUexh,
     &  8H  maxeff, 8H  Uratio, 8H  Am/Mbo, 8H   Am/Mi,
     &  9H   Mi/Mbo)
  103 FORMAT(2X, F3.1, F8.4, F8.4, E11.4, F8.4, F8.3, F8.4, F9.5,
     &  F8.2)
  104 FORMAT(2X, 8H UEXH = , E11.4, 7H EFF = , F20.4)
  105 FORMAT(/, 2X, 24HConversion Efficiency = , F3.1)
      DIMENSION UENDI(7)
      DATA UENDI/0.2, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0/
      OPEN(8, FILE = 'OPTUEXH.DAT', STATUS = 'OLD')
      DUMP = 1836.               !charge neutralization  1/31/97
    2 CONTINUE
```

```
      WRITE(*,101)
      READ(*,*) ETA              !conversion efficiency
      WRITE(8,105) ETA
      IF(ETA .EQ. 0.) GO TO 99
      WRITE(*,102)
      WRITE(8,102)
      DO 20 IU = 1, 7
      UEND = UENDI(IU)
      GAMEND = SQRT(1. + UEND*UEND)
      VEND = UEND/GAMEND
      THETA = LOG(UEND + SQRT(UEND*UEND + 1.))   !asinh
      UEXH = 0.01 + SQRT(DUMP*DUMP + 1.) - 1.
      UEXHN = UEXH
      EFF = 0.
      IOPT = 0
    1 CONTINUE
      UEXHNN = UEXHN
      UEXHN = UEXH
      UEXH = 1.01 * UEXH
      GAMEX = SQRT(1. + UEXH*UEXH)
      VEXH = UEXH/GAMEX
      VEXHP = UEXH/((GAMEX - 1.)/ETA + 1. + DUMP)  !charge neut
      RN = R
      R = 1.E4
      IF(VEXHP .GT. .05) R = EXP(THETA/VEXHP)
      EFFNN = EFFN
      EFFN = EFF
      EFF = (GAMEND - 1.) * (GAMEX - 1. + ETA * (1.+ DUMP))/
     & ((R - 1.) *  ETA * (GAMEX - 1.))
C     WRITE(*,104) UEXH, EFF
      OPTUEXH = 0.
      IF(EFF .LT. 1.00001*EFFN .AND. UEXH .GT. 0.1) THEN
          Y1 = EFF
          Y2 = EFFN
          Y3 = EFFNN
          X1 = UEXH
          X2 = UEXHN
          X3 = UEXHNN
          A = ((Y1-Y2)*(X2-X3)-(Y2-Y3)*(X1-X2))/
     &      ((X1*X1-X2*X2)*(X2-X3)-(X2*X2-X3*X3)*(X1-X2))
          B = ((Y1-Y2) - A*(X1*X1-X2*X2))/(X1-X2)
          OPTUEXH = -B/(2.*A)
          IF(OPTUEXH .GT. UEXH) OPTUEXH = UEXH
      END IF
      IF(OPTUEXH .NE. 0.) THEN
```

```
      GEXHOPT = SQRT(1. + OPTUEXH*OPTUEXH)
      OPTVEXH = OPTUEXH/GEXHOPT
      OPTVEXHP = OPTUEXH/((GEXHOPT - 1.)/ETA + 1. + DUMP)
      URATIO = UEND/OPTUEXH
      ROPT = EXP(THETA/OPTVEXHP)
      EFFMAX = (GAMEND - 1.) * (GEXHOPT - 1. + ETA
     &   * (1. + DUMP))/(ETA * (ROPT - 1.) * (GEXHOPT - 1.))
      AMRATIO = (GAMEND - 1.)/(2. * EFFMAX)
      URATIO = UEND/OPTUEXH
      AMMBO = (GAMEND - 1.)/(2.* EFFMAX * ETA)
      AMMI = (GAMEND - 1.)/(2.* EFFMAX * ETA * ROPT)
      WRITE(*,103) UEND, VEND, OPTVEXH, OPTUEXH, EFFMAX, URATIO,
     &           AMMBO, AMMI, ROPT
      WRITE(8,103) UEND, VEND, OPTVEXH, OPTUEXH, EFFMAX, URATIO,
     &           AMMBO, AMMI, ROPT
      IOPT = 1
    END IF
    IF(IOPT .EQ. 0) GO TO 1
 20 CONTINUE
    GO TO 2
 99 STOP
    END
```

```
PROGRAM TRIP                    !8/18, 9/6/96
  101 FORMAT(2X, 21H Acceleration (gs) = )
  102 FORMAT(2X, E13.4, F8.4)
  103 FORMAT(2X, F5.3, 4F8.4, F8.3, F7.3, F7.3, E11.4)
  104 FORMAT(2X, 21H Trip (Earth) Time = , F7.4)          !9/6
    DIST = 11.9
    XACC = 0.5 * DIST
    ETA = 1.
    F = 1.          !f = 1. for am; f = 261. for D + D + D
    ADDC = 1.
    IF(F .GT. 1.) ADDC = 0.
   2 CONTINUE
    WRITE(*,101)
    READ(*,*) AG
    IF(AG .EQ. 0.) GO TO 99
    ACC = 1.0324 * AG
    UEND = SQRT(ACC * XACC * (ACC * XACC + 2.))
    TAPP = 2. * UEND/ACC                              !9/6
    WRITE(*,104) TAPP                                 !9/6
    GAMEND = SQRT(1. + UEND*UEND)
    VEND = UEND/GAMEND
    THETA = LOG(UEND + GAMEND)          !asinh(Uend)
    TACC = THETA/ACC
    IF(F .GT. 1.) THEN
        XOPT = .09580
        GO TO 3
    END IF
    X1 = 0.05                       !X = VEXH
    X2 = X1
   1 CONTINUE
    X3 = X2
    X2 = X1
    X1 = X1 + 0.01
    GAMEX = 1./SQRT(1. - X1*X1)
    R = 1.01
    IF(X1 .GT. .05) R = EXP(THETA * (ADDC + (GAMEX - 1.) * F/ETA)/
   & (GAMEX * X1))
    Y3 = Y2                         !Y = EFF
    Y2 = Y1
    Y1 = (GAMEND - 1.) * (ADDC + (GAMEX - 1.) * F/ETA)/((R - 1.) *
   &  (GAMEX - 1.))
    IF(Y1 .LT. Y2 .AND. X1 .GT. 0.1) THEN
        A = ((Y1-Y2)*(X2-X3)-(Y2-Y3)*(X1-X2))/
   &      ((X1*X1-X2*X2)*(X2-X3)-(X2*X2-X3*X3)*(X1-X2))
```

```
        B = ((Y1-Y2) - A*(X1*X1-X2*X2))/(X1-X2)
        XOPT = -B/(2.*A)
        GO TO 3
      END IF
      GO TO 1
    3 CONTINUE
      OPTVEXH = XOPT
      OPTUEXH = OPTVEXH/SQRT(1. - OPTVEXH*OPTVEXH)
      GEXHOPT = SQRT(1. + OPTUEXH*OPTUEXH)
      ROPT = EXP(THETA * (ADDC + (GEXHOPT - 1.) * F/ETA)/(GEXHOPT *
     &  OPTVEXH))
      EFFMAX = (GAMEND - 1.) * (ADDC + (GEXHOPT - 1.) * F/ETA)/
     &  ((ROPT - 1.) * (GEXHOPT - 1.))
      URATIO = UEND/OPTUEXH
      AMMBO = (GAMEND - 1.) * F * (2. - ADDC)/(2.* EFFMAX * ETA)
      AMMI = AMMBO/ROPT
      WRITE(*,103) UEND, VEND, OPTVEXH, OPTUEXH, EFFMAX, URATIO,
     &            AMMBO, AMMI, ROPT
      ROVER = ROPT * ROPT
      FRAM = AMMBO/(ROPT - 1.)
      OVAMMBO = FRAM * (ROVER - 1.)
      USE = OVAMMBO/51.47
      TTRIP = 2. * TACC
      WRITE(*,102) USE, TTRIP
      GO TO 2
   99 STOP
      END
```

```
   PROGRAM COPOBM                          !9/7/96
101 FORMAT(2X, 21H Initial Accel (g) = )
102 FORMAT(1X, 6H Tship, 3X, 6H Theta, 5X, 5H Dist, 2X,
  & 9H Prop Vel, 3X, 6H Accel, 2X, 7H TEarth, 2X, 6H Temit)
103 FORMAT(3X, F4.1, 3X, F6.4, 3X, F7.4, 4X, F7.4, 3X, F6.4,
  & 3X, F6.4, 2X, F6.4)
  1 CONTINUE
   WRITE(*,101)
   READ(*,*) AGO
   IF(AGO .EQ. 0.) GO TO 99
   AO = 1.0324 * AGO
   WRITE(*,102)
   DO 10 IT = 1, 11
   FT = IT - 1
   TIM = 0.5 * FT
   ARG = AO * TIM + 1.
   THET = LOG(ARG)
   DIST = 0.5 * (0.5 * AO * TIM*TIM + TIM - THET/AO)
   PVEL = 0.5 * (ARG - 1./ARG)
   ACC = AGO/ARG
   TAPP = 0.5 * (0.5 * AO * TIM*TIM + TIM + THET/AO)
   TEM = TAPP - DIST
   WRITE(*,103) TIM, THET, DIST, PVEL, ACC, TAPP, TEM
 10 CONTINUE
   GO TO 1
 99 STOP
   END
```

```
      PROGRAM SAILTRIP                        !9/17/96
  101 FORMAT(3X, 12HParameter = )
  102 FORMAT(2X, 5HTship, 4X, 5HTheta, 6X, 4HDist, 4X,
     & 8HProp Vel, 5X, 5HAccel, 4X, 6HTEarth, 4X, 5HTemit, 2X,
     & 6HMratio)
  103 FORMAT(1X, F6.4, 3X, F6.4, 3X, F7.4, 4X, F8.4, 3X, F7.4,
     & 3X, F7.4, 2X, F7.4, F8.4)
  104 FORMAT(2X, 19H Exhaust Velocity =, F8.5,
     & 14H; Proton MeV =, F7.1)
  105 FORMAT(3X, 17HFinal sail furl =, F8.3)
      ETA = 1.
      AGO = 1.
      AO = 1.0324 * AGO
      DSTAR = 11.9
      DTA = 0.01
      DTS = 0.001
      DTP = 0.5
    1 CONTINUE
      WRITE(*,101)
      READ(*,*) PAR
      IF(PAR .EQ. 0.) GO TO 99
      ETA = PAR
C.....Acceleration Phase
      IT = -1
      TIM = -DTA
      ACC = AGO
      WRITE(*,102)
    2 CONTINUE
      IT = IT + 1
      FT = IT
      TIMN = TIM
      TIM = DTA * FT
      ITSN = TIMN/DTP
      ITS = TIM/DTP
      IF(TIM .EQ. 0. .OR. ITS .NE. ITSN) THEN
        FITS = ITS
        TIMI = DTP * FITS
        ARG = AO * TIMI + 1.
        THETI = LOG(ARG)
        DISTI = 0.5 * (0.5 * AO * TIMI*TIMI + TIMI - THETI/AO)
        UI = 0.5 * (ARG - 1./ARG)
        AGI = AGO/ARG
        TEI = 0.5 * (0.5 * AO * TIMI*TIMI + TIMI + THETI/AO)
        TEMI = TEI - DISTI
```

```
        FMR = 1.
        WRITE(*,103) TIMI, THETI, DISTI, UI, AGI, TEI, TEMI, FMR
      END IF
      THET = LOG(AO * TIM + 1.)
      DISTN = DIST
      DIST = 0.5 * (0.5 * AO * TIM*TIM + TIM - THET/AO)
      IF(DIST .GT. 0.5*DSTAR) THEN
        DRAT = (0.5 * DSTAR - DISTN)/(DIST - DISTN)
        TST = TIMN + (TIM - TIMN) * DRAT
        ARG = AO * TST + 1.
        THETAT = LOG(ARG)
        DISTT = 0.5 * (0.5 * AO * TST*TST + TST - THETAT/AO)
        UT = 0.5 * (ARG - 1./ARG)
        AGT = AGO/ARG
        TET = 0.5 * (0.5 * AO * TST*TST + TST + THETAT/AO)
        TEMT = TET - DISTT
        FMT = 1.
        WRITE(*,103) TST, THETAT, DISTT, UT, AGT, TET, TEMT, FMT
        GO TO 10
      END IF
      GO TO 2
C.....Deceleration Phase
   10 CONTINUE
      IPRN = 0
      ND = 0
      VEXH = 0.9
   11 CONTINUE
      ND = ND + 1
      IF(ND .EQ. 2) VEXH = 0.85
      GEXH = 1./SQRT(1. - VEXH*VEXH)
      RELF = (GEXH * VEXH)/(GEXH - 1.)
      TS = TST
      TE = TET
      FM = FMT
      THETA = THETAT
      X = 0.
      ACC = 1.0324 * AGT
      FURL = 1.
   12 CONTINUE
      U = SINH(THETA)
      GAM = COSH(THETA)
      XN = X
      X = XN + U * DTS
      PR = FURL * FMT * AO * EXP(-THETA)
      TBM = PR
```

```
 TEX = ETA * PR * RELF
 FMN = FM
 FM = FMN - TEX * DTS/(GEXH * VEXH)
 TSN = TS
 TS = TSN + DTS
 TEN = TE
 TE = TEN + GAM * DTS
 ACCN = ACC
 ACC = (TBM - TEX)/FMN
 AGE = (ACC + (ACC - ACCN))/1.0324
 IF(AGE .LT. -1.) FURL = -FURL/AGE
 THETAN = THETA
 THETA = THETAN + ACC * DTS
 ITSN = TSN/DTP
 ITS = TS/DTP
 IF(ITS .NE. ITSN) THEN
   FITS = ITS
   TSI = DTP * FITS
   TSIR = (TSI - TSN)/(TS - TSN)
   THETAI = THETAN + (THETA - THETAN) * TSIR
   UI = SINH(THETAI)
   XI = XN + (X - XN) * TSIR + DISTT
   AGI = (ACCN + (ACC - ACCN) * TSIR)/1.0324
   TEI = TEN + (TE - TEN) * TSIR
   TEMI = TEI - XI
   FMI = FMN + (FM - FMN) * TSIR
   FMR = FMT/FMI
   IF(IPRN .EQ. 1) WRITE(*,103) TSI, THETAI, XI, UI, AGI,
&    TEI, TEMI, FMR
 END IF
 IF(THETA .LT. 0.) THEN
   THETR = THETAN/(THETAN - THETA)
   TSF = TSN + (TS - TSN) * THETR
   THETAF = 0.
   DVEXH = VEXHP - VEXH
   VEXHP = VEXH
   DISTFP = DISTF
   DISTF = XN + (X - XN) * THETR + DISTT
   IF(ND .GT. 1 .AND. IPRN .EQ. 0) THEN
     VEXH = VEXHP + (DSTAR-DISTF) * DVEXH/(DISTFP-DISTF)
     IF(ABS(DISTF-DSTAR) .LT. 0.0001) THEN
       IPRN = 1
       GEXH = 1./SQRT(1. - VEXH*VEXH)
       PMEV = 938. * (GEXH - 1.)
       WRITE(*,104) VEXH, PMEV
```

```
      END IF
      GO TO 11
     END IF
     PVEL = 0.
     AGF = (ACCN + (ACC - ACCN) * THETR)/1.0324
     TEF = TEN + (TE - TEN) * THETR
     TEM = TEF - DISTF
     FMF = FMN + (FM - FMN) * THETR
     FMRAT = FMT/FMF
     IF(IPRN .EQ. 1) THEN
      WRITE(*,103) TSF, THETAF, DISTF, PVEL, AGF, TEF, TEM,
    &     FMRAT
      WRITE(*,105) FURL
      GO TO 1
     END IF
     GO TO 11
    END IF
    GO TO 12
99 STOP
    END
```